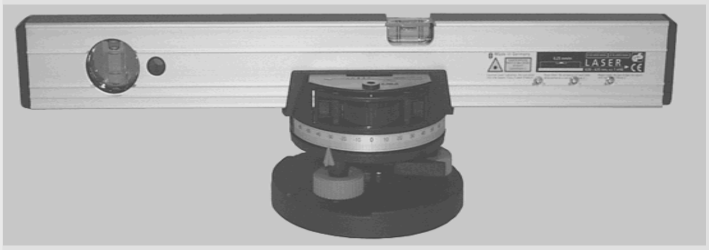


# Experiment 22

# Laser Levelling


MEOS

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## 1.0 Introduction

When Javan and his colleagues discovered the first laser in the year 1960, it was considered as an academic curiosity. Today it is difficult to encompass the bandwidth of the applications of lasers in a few words. The cutting of several cm thick steel plates, the construction of structures with an accuracy in the realm of nanometer, demonstrate very distinctly the lasers multiple possibilities while working on materials. The property of lasers used here is the high radiation power, which by the focusing of the beam diameter to low some mm leads to extremely high intensities. Of the many, the most frequent applications of the laser are not based on its radiation power, but rather on its focusing capability.

The words that you are reading here, have been printed by a Laser printer. While reading these lines, you may prefer to hear the playback of a CD, with the realisation that here also the laser plays a very important role, with the focusing capability of the laserdiode of the radiation be-ing foremost. When one sees the numbers of lasers produced yearly, the figures of laserdiodes produced, far exceed the other numbers by a large margin. An equally large field of application of laserdiodes is in the News Media technique where the transmitted information con-sisting of modulated laser light is transported thousands of kilometres by the use of optical fibres. It should be kept in mind that as soon as an improved laser is developed in the production of laser diodes, it enters the cycle of further refinements.

Another not so common application of the laser is found in measurement techniques. These applications utilise the high monochromatic property of the laser, with the precise definition of the wavelength as a tool for the measurement of length. One measures an object in units from the wavelength which are translated to a definite correlation on the measuring meter. A basic type of such measuring instrument is the Michelson Interferometer which utilises the interference capability of coherent laser light.

There exist a whole series of measurement problems which cannot be resolved by the above mentioned methods. Typical characteristic problems of measurement tasks is the comparatively large distance to the object being examined. Michelson Interferometers cannot be used in this case, because they are based on a more or less precise linear movement of the measuring re ectors. Above all, it is important that the re ector is lead from a zero point till the object without interruption of the laser beam. For further convenience, a method has been developed by which the transit time of a laser pulse to the object and back can be measured. In principle, this technique by itself is not new. It has been used since long for Echo Sound Measurement with Ultrasonics or in Radar technique. Whether Ultrasonics or radio waves can be collimated to a low divergent beam as it can be done with lasers.

The purpose is mostly to determine the distance to a possibly sharply localised object. For example; when the distance to a church tower top or the distance to the moon has to be actually measured with the laser.

But there are also a lot of other simple applications where the straightness of the laser beam plays the important role.

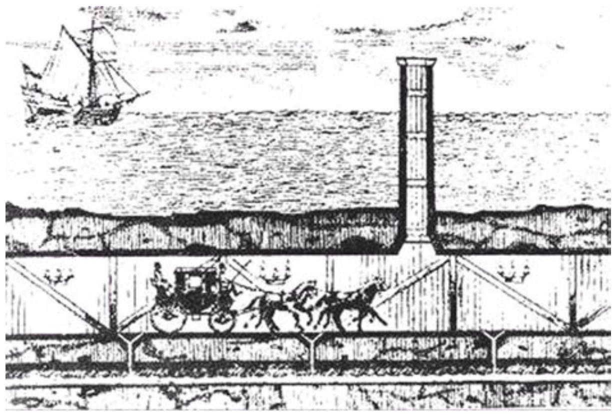

**Fig. 1:** One of the earliest plans (1802) was that of Albert Mathieu Favier, a French engineer who proposed a tunnel for horse-drawn vehicles. An artificial island in the middle of the Channel would provide a stage post for changing the horses; the tunnels‘ lighting was to be provided by oil lamps and chimneys would provide ventilation.

It took more than 200 years before the first ideas of the channel tunnel connecting Europe to Great Britain became reality. Thanks the laser levelling technique both the British and French team met at the right place with a deviation of a few centimetres on a distance of 50 km only.

### *1.1 Basics in Levelling*

Since people started to erect buildings they have been faced with the problem of levelling. Before the building could be erected a plain area horizontal to the earth had to be defined and prepared. For huge buildings like the pyramids we know today that the ancient Egyptians used an artificial lake to define and prepare such an area. In the years of 500 BC the spirit level has been invented which nowadays is still in use and belongs to the most important tools. The combination of a spirit level and a telescope lead to the first levelling instruments around 1870.

**Fig. 2: Levelling Instrument from 1870**

By means of such an instrument and a levelling pole or staff straight lines and plain areas can be defined and measured. The operation requires at least two persons, one person is looking through the instrument while the other person holds the levelling pole. By using an additional laser, the measuring process can be handled now by one person only since the laser line once aligned remains. Nowadays skilled worker are using laser levelling instruments not only for construction of a building but also for levelling a lot of other interior. For this purpose each oor of

a building is equipped with one or more height reference points which are used by the worker to transfer the reference by means of their own laser levelling instruments to places where they are needed.
The following chapters are addressed to people who will be involved in the development of laser assisted levelling instruments rather than to use it.

## 1.2 Properties of a Laser Beam

Real parallel light beams do not exist in reality and plane wave fronts exist only at a particular point. The reason for the failure of geometrical optics is the fact that it has been defined at a time where the wave character of light was still as unknown as the possibility to describe its behaviour by Maxwell‘s equations. To describe the propagation of light we use the wave equation

$$\Delta\vec{E} - \frac{n^2}{c^2} \cdot \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

When we consider the technically most important case of spherical waves propagating in the direction of z within a small solid angle as the Laser actually does, we arrive at the following statement for the electrical field:

$$\vec{E} = \vec{E}(r, z)$$

with

$$r^2 = x^2 + y^2 + z^2$$

In this case the solution of the wave equation provides fields which have a Gaussian intensity distribution over the cross section. Therefore they are called Gaussian beams. Such beams, especially the Gaussian fundamental mode ($TEM_{00}$) are generated with preference by lasers. But the light of any light source can be considered as the superposition of many such Gaussian modes. Still, the intensity of a particular mode is small with respect to the total intensity of the light source. The situation is different for the laser. Here the total light power can be concentrated in the fundamental mode. This is the most outstanding difference with respect to ordinary light sources next to the monochromasy of laser radiation. Gaussian beams behave differently from geometrical beams.

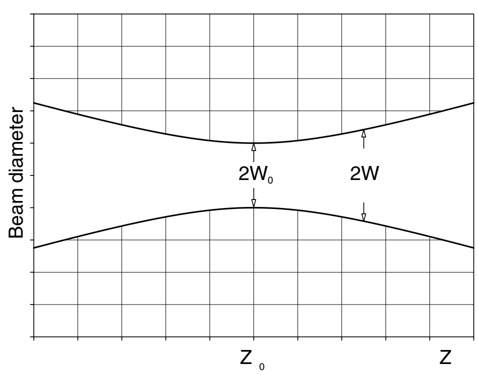


**Fig. 3: Beam diameter of a Gaussian beam as fundamental mode $TEM_{00}$ and function of z.**

A Gaussian beam always has a waist and its radius w results out of the wave equation as follows:

$$w(z) = w_0 \cdot \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

$w_0$ is the smallest beam radius at the waist and $z_r$ is the Rayleigh length. In Fig. 3 the course of the beam diameter as a function of z is represented. The beam propagates within the direction of z. At the position z = $z_0$ the beam has the smallest radius. The beam radius increases linearly with increasing distance. Since Gaussian beams are spherical waves we can attribute a radius of curvature of the wave field to each point z. The radius of curvature R can be calculated using the following relation:

$$R(z) = z + \frac{z_r^2}{z}$$

This context is re ected by fig. 4. At z = $z_r$ the radius of curvature has a minimum . Then R increases with 1/z if z tends to z = 0 . For z=0 the radius of curvature is infinite. Here the wave front is plane. Above the Rayleigh length $z_r$ the radius of curvature increases linearly. This is a very essential statement. Due to this statement there exists a parallel beam only in one point of the light wave, to be precise only in its focus. But within the range

$$-z_r \leq z \leq z_r$$

a beam can be considered as parallel or collimated in good approximation.

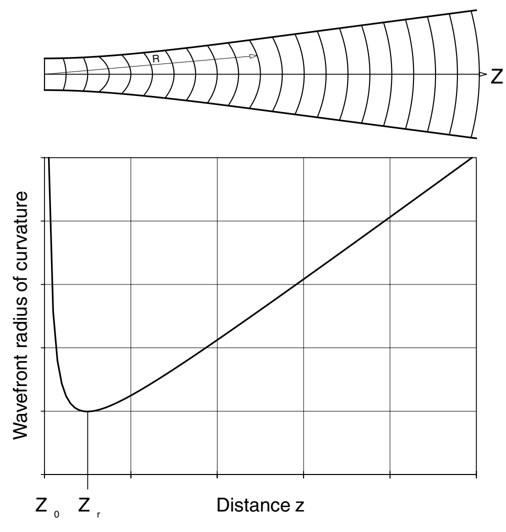


**Fig. 4: Course of the radius of curvature of the wave front as a function of the distance from the waist at z=0**

In fig. 5 the Rayleigh range has been marked as well as the divergence θ in the far field, that means for z>>$z_0$. The graphical representation does not well inform about the extremely small divergence of laser beams another outstanding property of lasers.
The reason for this is that the ration of the beam diameter with respect to z has not been normalised. Let‘s consider, for example, a HeNe-Laser (632 nm) with a beam radius of $w_o$=1mm at the exit of the laser. For the Rayleigh range we get:

$$2 \cdot z_R = 2w_0^2 \frac{\pi}{\lambda} = 2 \cdot 10^{-6} \frac{3.14}{623 \cdot 10^{-9}} = 9{,}9 \text{ m}$$

That means that within a range of nearly 10 m the beam can be considered as parallel.

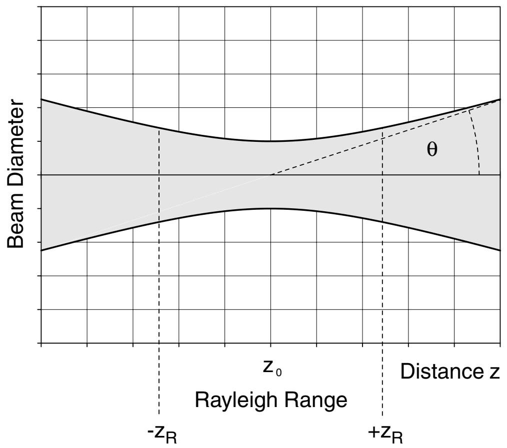


**Fig. 5: Rayleigh range $Z_R$ and divergence θ for the far field $z>>z_R$**

### *1.3 Distance Measurement*

The laser levelling instruments helped the skilled worker to reduce the number of involved employees for levelling tasks. But when it comes to verify the proper dimensions of their work, they still needed an extra person to perform the measurements. Nowadays one will find in the storage case for the measurement instruments of a skilled worker a laser distance finder. Therefore this project has also been equipped with such an instrument which fundamentals will be given in the next chapters.
Depending on the range to be measured two different kinds of measurements are used. For distances above several ten metres the time of ight principle is used whereas below this range of 0,3 to 30 metres the phase shift technique is applied.

### *1.3.1 Time of ight measurement*

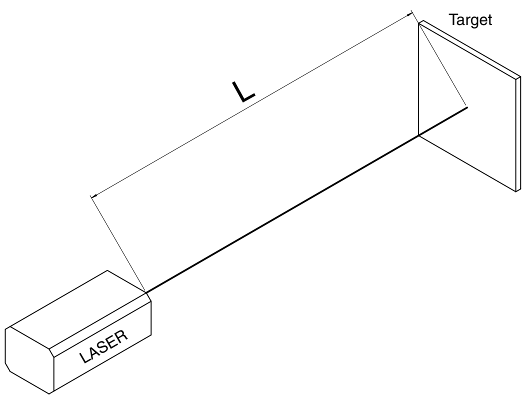


**Fig. 6: Principle of transit time measurement**

At the distance L, we find the object to be measured. The time Δt that a laser pulse requires in order to return from the object to the laser is:

$$\Delta t = 2 \cdot \frac{L}{v} = 2 \cdot \frac{n \cdot L}{c}$$

Where v is the speed of the laser pulse, n the refractive index of the surrounding air and c is the speed of light in a vacuum. The refractive index of air is mainly determined through its density, which on the other hand is dependent on air pressure, temperature and its composition. The common formula for the calculation of the refractive index of air as a function of the air pressure P (in hPa) and the temperature υ (in °C) originates from Edlén and has become the standard for dry air:

$$(n-1)_{p,\upsilon} = 2.8775 \cdot 10^{-7} \cdot P \cdot \frac{1 + 10^{-6} \cdot P \cdot (0.613 - 0.00997 \cdot \upsilon)}{1 + 0.003661 \cdot \upsilon}$$

Water content in air has an in uence on the refractive index n. The relative humidity RF (in %) leads to the reduction of the refractive index and thus for standard humid air, it results in an additional term:

$$\Delta (n-1)_{RF} = -3.03 \cdot 10^{-9} \cdot RF \cdot e^{0.057627 \cdot \upsilon}$$

The derivation of the refractive index thus depends on the following parameters:

$$\frac{dn}{d\upsilon} = -0.93 \cdot 10^{-6} \left[K^{-1}\right]$$

$$\frac{dn}{dP} = +0.27 \cdot 10^{-6} \left[hPa^{-1}\right]$$

$$\frac{dn}{dRF} = -0.96 \cdot 10^{-8} \left[\%^{-1}\right]$$

An error in estimation results from the total of the differential relationship for the determination of the distance L.

$$L = \frac{c}{2 \cdot n} \cdot \Delta t$$

the relative error:

$$\frac{dL}{L} = \left|\frac{dn}{n}\right| + \left|\frac{d\Delta t}{t}\right|$$

The speed of light in a vacuum as per the definition is error free. With the above the relative error for the determination of the distance with different values for air pressure, air temperature, the relative humidity and the precise time measurement can be calculated for the respective measuring task.

After these fundamental considerations, the construction according to Fig. 6 should now be improved progressively so as to achieve in the end a technically meaningful instrument.

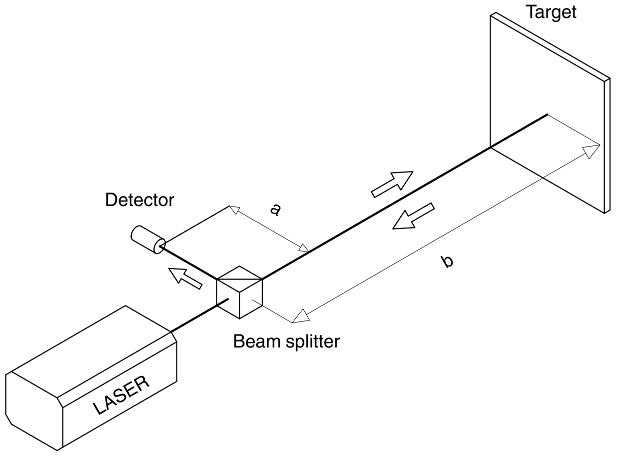


**Fig. 7: Transit time measurement with detector**

In practice, a construction according to Fig. 7 is used. Here one sets up the total length L as the sum of the paths a and b. The path a is in general a set-up constant which then merits consideration only if b is not very larger as a. The laser beam traverses through a beam splitter which alters the output corresponding to the dividing proportion. A part of the returning laser beam hits the detector. As there is no ideal beam splitter, a part of the laser beam in the passage through the beam splitter also comes in contact with the detector. One therefore gets the initial starting impulse for time measurement. Assuming the refractive index of the atmosphere to be 1 and the distance to the target object to be 1 m, one derives the transit time of Dt from:

$$\Delta t = \frac{2}{3 \cdot 10^8} = 6.7 \text{ nsec}$$

In order to achieve a separation of the initial starting impulse and the echo impulse at the detector, the laser pulse must be smaller than this value. The laser should be able to emit short light pulses in the nsec region with an adjustable repetition rate. The required output depends on the respective application area.

### 1.3.2 Phase shift method

When the laser emission $I_0$ is modulated in its intensity, one creates a new "synthetic" wave.

$$I = I_0 \cdot \sin(\nu \cdot t)$$

This new wave has a wavelength $\lambda$ according to:

$$\nu = \frac{c}{\lambda} \rightarrow \lambda = \frac{c}{\nu} = c \cdot \tau ,$$

whereby c is the speed of light, $\tau$ denotes the period and $\nu$ the modulation frequency. If we are using a period of 500 nsec or a frequency of 2 MHz the wavelength is 150 m. To obtain the distance travelled of this new wave we have to compare the phase of the outgoing with the incoming intensity.

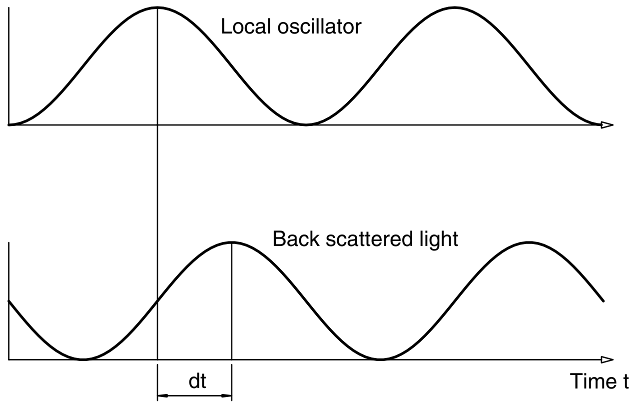


**Fig. 8: Phase shifted signals**

Instead of using the intensity of the outgoing wave we can also use directly the signal of the modulation source, the local oscillator. This saves one photo detector. The phase shift is related to the time which the new wave needed to reach the target and to come back. From dt we obtain the travelled distance to:

$$\frac{dt}{\tau} = \frac{ds}{\lambda} \rightarrow ds = \lambda \cdot \frac{dt}{\tau} = c \cdot dt$$

and the distance L to the target as:

$$L = \frac{1}{2} ds$$

One disadvantage of this method is, when the distance of the target exceeds 75 m (the half of the synthetic wavelength) lets say 80 m, the system would give us a value of 5 m. Another problem will be the detection of small phase shifts, since the back scattered light from the target has a fairly low and therefore noisy intensity (see also the next chapter). Modern systems are therefore using two different modulation frequencies. For systems optimised for a detection range of 0.3 to 30 m are using frequencies with 60 and 500 nsec period. The synthetic wavelength of the shorter one is 18 m and the other one as we already know 150 m. By using a microprocessor the first wave with 18 m is launched and detected and subsequently the 150 m long one. For each wavelength the procedure is repeated until the deviation lies within the specified tolerance. After that the processor compares both results. If the short wave reports let us say 3 m, the real result can be 3, 9+3, 18+3 and so on. If the long wave reports 12 m, the real result can be 12, 75+12, 150 +12 and so on. To find out which result is unambiguously we use the relation for the distance L:

$$L = n \cdot \frac{\lambda}{2} + \varepsilon$$

The letter n stands for an integer and $\varepsilon$ for the measured value. Is for example n equal to one, then the distance L lies within the second period. Since both measurements must yield the actual distance L the following expression must be fulfilled:

$$L = n_s \cdot \frac{\lambda_s}{2} + \varepsilon_s = n_L \cdot \frac{\lambda}{2} + \varepsilon_L$$

The index S stands for the short and L for the long wavelength. With the known values this expression becomes

$$n_s \cdot 9 - n_L \cdot 75 = 9$$

and can only be resolved for $n_S = 1$ and $n_L = 0$. Therefore the unambiguously result for L is 12 m.

### 1.4 Considerations for the receiver optics

The desired distance to the target object as well as the back scattering properties of the object play an important role. Now prior to the final selection of the laser, an immediate estimation is to be made of the quantity of available light scattered back from the target object. In this context, the absorption loss in the measured length as well as the sensitivity of the detector should be taken into consideration. Indeed, the estimation of the returning scattered light from the target object should be first considered. Without any doubt, this value depends on the properties of the target object. However, it is possible to compare the back scattering behaviour for many target surfaces with that of the Lambert‘s radiators. Such a type of radiator emits the received light according to the cosine distribution, so that it can be designated as a Cosine radiator. (Fig. 9).

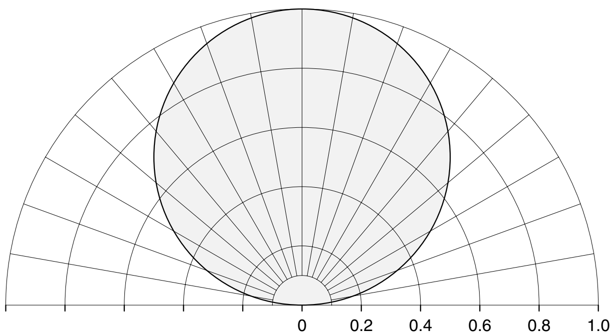


**Fig. 9: Lambert‘s Cosine radiator**

The main direction of the radiation is perpendicular to the surface of the object. Whenever possible one must therefore target the object in such a way, that this direction coincides with the measurement axis of the range meter. In reality the object distance L would be very large compared to the aperture of the telescope. Therefore one can consider the radiator as emitter of spherical waves whose intensity decreases with the quadrate of the distance L and according to the cosine distribution. The power which finally reaches the photo detector is after all, just the power which enters through the area of the input lens. The exact formulation results in:

$$P = \beta \cdot P_0 \cdot \int_{d\Omega} \cos(\vartheta) \cdot d\Omega$$

In which β is an constant which characterises the re ectivity of the object. With the sufficiently large distance, the input lens receives almost only light from a radiator, which originates within the angle $\upsilon \approx 0$. Consequently, the above equation is simplified to:

$$P = \beta \cdot P_0 \cdot \int_{d\Omega} d\Omega = \beta \cdot P_0 \cdot \Delta\Omega$$

The angle ΔΩ is given by the solid angle of the telescope in which the radiation is picked up.

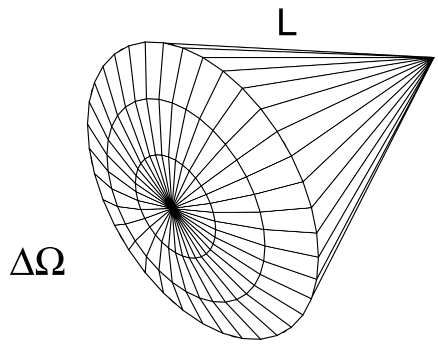


**Fig. 10: Solid angle of received radiation**

The radiator (target) emits radiation only in a half-sphere as it does not radiate from the rear. Therefore the ratio of the spherical segment area of the telescope to the surface of the half-sphere is directly proportional to the ratio between the received output and the radiated output:

$$\frac{2 \cdot \Delta\Omega}{\Omega} = \frac{F_{lens}}{2 \cdot \pi \cdot L^2} = \frac{P}{\beta \cdot P_0},$$

or

$$P = \beta \cdot P_0 \cdot \frac{F_{lens}}{2 \cdot \pi \cdot L^2} = \frac{\beta}{2} \cdot P_0 \cdot \frac{r^2}{L^2}$$

The received power P decreases to the quadrate to the distance L of the target object and it increases with the increase of the lens diameter r of the receiver optics.
With this knowledge we are prepared to start the design of laser range finder. The initial power $P_0$ is determined by the laser safety regulations. To stay within the class 2 and when using radiation around 635 nm for a given smallest beam diameter the power thus is determined. To match the system one can control the aperture of the receiver and the selection of sensitive photo detectors.

## 2.0 Experimental Equipment

### 2.1 Laser Levelling

The laser levelling system consists of the tripod, adjustable base plate and the spirit level with integrated diode laser emitting 1 mW at 635 nm. Additional accessories are a rotary 90° beam bender, target screen and laser vision goggles. The goggles ar no laser safety goggles, since these are not required for a class 2 laser, however, they are used to enhance the visibility the laser spot. The additional target screen can be used, when the object under consideration does not produce sufficient back scattered light.
The complete equipment except the tripod are coming in a storage and transportation box.