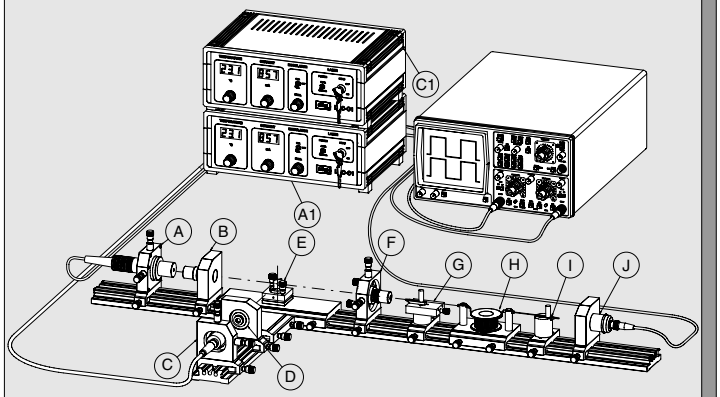
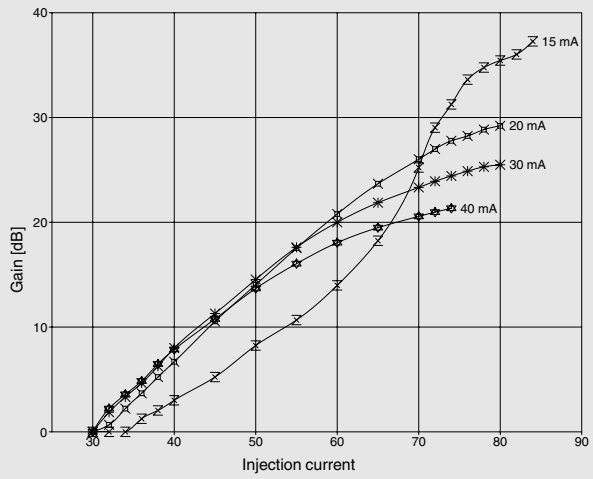
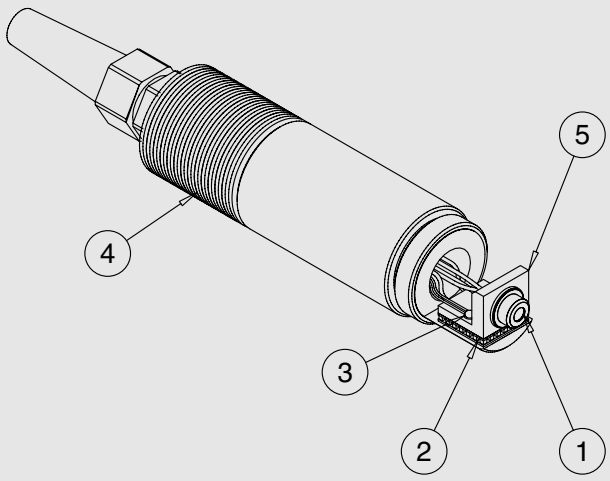



# EXPERIMENT 14



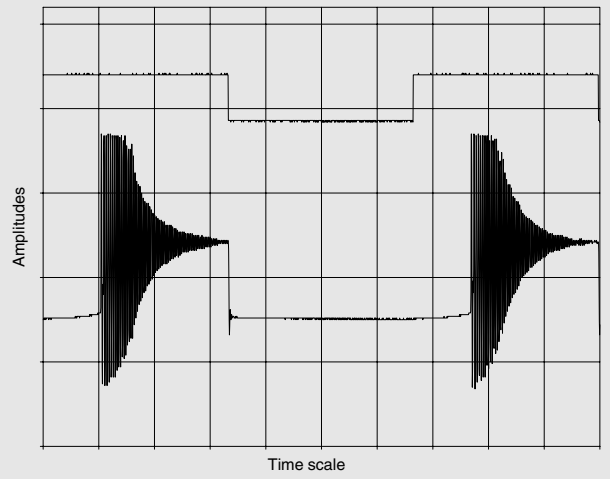

# Erbium doped Fibre Amplifier




MEOS

Didactic Counsellor

Prof. Dr.-Ing. Dickmann




Fachhochschule Münster

Fachbereich Physikal. Technik

1 INTRODUCTION 3

2 BASICS 4

2.1 Interaction of light and matter 4

2.2 Natural line width 5

1.3 Homogeneous line broadening 7

1.4 Two level system 7

1.5 Optical Amplifier 8

1.6 Three-level system 8

3 EXPERIMENTS 11

3.1 Experimental set-up, Description of modules 11

3.2 Experimental set-up, step 1 16

3.3 Experimental set-up, step 2 17

3.4 Experimental set-up, step 3 18

3.5 Experimental set-up, step 4 19

3.6 Experimental set-up, step 5 19

3.7 Experimental set-up, the final step 20

3.8 Fibre Laser 22

4 LASER SAFETY 24

# 1 Introduction

These days as one reaches out for the phone to call somebody up, one hardly realises the great technological change that has taken place during the past decade in the field of Telecommunications. This is also a characteristic of the epochal change from an industrial society to a communication society.
The globalisation of the markets require, essentially more information on the one hand, for the development and production and on the other, more communication with distant markets. There is also a steady increase in communication requirements even in the private areas due to the possibility of being part of world-wide Datanets for e.g. the Internet with the help of a simple PC. All this led to a rapid increase in the number of subscribers to the Telecommunication network and that necessitated a markedly bigger transmission bandwidth. The transmission speed had to be likewise drastically increased since the largest part of communication occurred between computers.
The copper cables that were used up to now, however are overtaxed in the face of this challenge. It is true that special cables with the required bandwidths can be produced, but due to the cost involved they are not an economical alternative to the optical waveguides which are replacing copper cables to an increasing degree, so much so that even newly laid down Nets are being equipped with them.
The idea of sending light signals by waveguides and making use of it in Data Transmission was predicted in 1939 by H. Buchholz in his publication "The Quasioptic of Ultrashortwave conductor". However it was only in the year 1962 with the development of the first semiconductor laser, that the serious technical realisation of Data transmission began, using this laser and glass fibre through which laser light was conducted. Thus, simple sources of light production and modulation, became available.

Today, information transmissions through laser diodes and glass fibres are the order of the day and the developments in this area belong to the most important ones in this century. This technology is based on the already known fundamentals so that no new understanding needs to be imparted. In practice, technically, however it means new challenges.
In the fibres used today, light is conducted within a core diameter of only 5 µm and for that on the one hand the fibre production and on the other, the fine mechanical components which are necessary for the launching of the light and for the installation of the fibres, needs to be developed. Further aims are the reduction in the transmission losses, internal fibre optical amplifiers should further reduce the number of electronic relays; and even laser diodes with smaller bandwidth that increase the transmission speed, to name a few.

A further important milestone in the development is the realisation of optical amplifiers that have properties which were previously achievable at high cost or did not exist at all in electronic amplifiers. Within a period of only 3 years of research and development, the optical amplifiers have revolutionised the future of glass fibre nets. The main reason for the replacement of the electronic with the optical amplifier lies therein that they can simultaneously amplify any data format and –rates within a comparatively extremely large spectral area. With that, the barrier of the small and limited bandwidth of electronic semiconductor amplifiers was broken. Out of the multitude of concepts for optical Amplifiers, the Erbium doped Fibre Amplifier (EDFA Erbium doped fibre amplifier) has turned out to be especially well suited.

The fundamental principle of this new technology is the production of an amplifying medium through optical pumping. The transport of information through electrical conductors results from a temporal change in a flow of electrons. Due to losses in the conductors, it become necessary to insert an amplifier after a specific conductor length in order to increase the electron count again. In Glass Fibres, the information transmission results through the temporal change in a flow of photons, whose count has to be likewise increased after a specific transport length.
Till now, this was done through the conversion of a stream of photons into a stream of electrons with the help of photodiodes with connected electronic amplification and the re-conversion of electrons into photons with laserdiodes. In optical amplifiers, however, this intermediate step is dropped; here the photon count is directly increased.
The theory and practice of optical amplifiers is as old as the laser technique itself, since Laser is nothing else but an optical amplifier that is increased as oscillation through feed-back. When one removes the feed-back (i.e. the cavity mirror) in a Laser, one gets an optical amplifier which is in a position to amplify a stream of photons if its frequency lies within the amplification band width of the optical amplifier.

The aim of this project is the introduction to the concept of optical amplification as they are employed for modern Telecommunication. This project also includes the Fundamentals of the production of optical amplification by means of optical pumping which is also the basis for optically pumped Laser-systems.
The discussion of diode lasers, which are used on the one hand as photon sources for information production and on the other as pumping light sources, supplements the technical knowledge.
The dependence of emission of Laserdiodes on the temperature and injection current are constituents of the measurements in the frame-work of the testing method. For the technical application as well as for the measurements within this project, photodiodes are necessary, whose characteristic features will be presented.

The fundamentals of optical wave guides, the launching of light and its transport are the more important constituents of the fundamentals, even more than the subject of the experiments. The application of fibre couplers will be avoided due to didactic reasons and dichroic elements will be used instead.



Dr. W. Luhs **MEOS** GmbH 79427 Eschbach - August 1998/ July 2003

# 2 Basics

## 2.1 Interaction of light and matter

The saying “to shed light upon a matter” means, more or less, to “illuminate” facts that are unclear. In 1644 Rene Descartes published his metaphysical ideas on the essence of light. Since then, people have been trying to shed light upon “light” itself.
According to his ideas, light consists of scattered particles which have different speeds in different bodies.
In 1667, R. Hooke claimed that this was all nonsense. He was the first person who thought that light consisted of quick oscillations.
Huygens demonstrated light ether in 1690. In 1717 Newton proved that light has a transversal quality. At that time, however, people could only imagine longitudinal waves, so Newton rejected the wave theory of light completely. Newton’s authority over the subject prevented the formulation of the wave theory of light for 100 years.
Unaffected by the dispute over the essence of light, James Clerk Maxwell summarised the electrical and magnetic appearances in a system of mathematical equations. In 1856, when Kohlrausch and Weber found out through measurements that the speed of electromagnetic waves was the same as that of light, Maxwell came to the conclusion that light is an electromagnetic oscillation. In 1888 Heinrich Hertz was able to give experimental proof of electromagnetic waves.
As can be easily imagined, due to the various interpretations on the nature of light it took a long time for the electromagnetic theory to be recognised as a basis for the sum of the physical experiences which could not be reduced any further. But, as we now know, even this theory has its limitations. It is possible to explain all appearances which occur in light scattering using this theory. However, it fails in the case of the emission and absorption of light.
Max Planck was able to solve the problems in this area with his formula $E = h\nu$. According to this formula light possesses both properties, i.e. corpuscular as well as wavelike qualities. This paradoxical formula could finally be clarified through quantum mechanics. There was a further change in classic optics in the sixties of this century when lasers were discovered.
For the first time, light was subjected to an unusually high intensity. People observed appearances such as the optical frequency doubling which led to the formulation of non-linear optics.
In classic, i.e. linear optics, the scattering of light in matter is described by both optical constants dependent on the frequency, the refractive number n and the absorption coefficient $\alpha$. In present linear optics, these variables are independent of the intensity of the light. Reflection, refraction, scattering, speed and absorption of light are therefore constants of the relevant medium and are not dependent on the light intensity. This resulted in two important principles used everywhere in optics: The superimposition principle (interference) and maintaining the frequency. Both these conditions are only valid in relatively small light intensities as can be obtained from normal light sources. Neither the superimposition principle nor the conservation of frequency apply to the high intensities of lasers. Therefore, linear optics is only specifically applicable to small light intensities.
The appearances observed on the basis of the interaction between light and matter can, in principle, be divided into two groups.

A. resonant phenomena
B. non-resonant phenomena

In the case of resonant light the incoming light has an energy of $E = h\nu$, which corresponds to the energetic distance of a transition. Electrons of the atoms or molecules in their initial state are transferred to $E_1$ which is in an excited state.
In the example of non-resonant light, the energy of the incoming light is much smaller than the energetic interval of the considered transition.

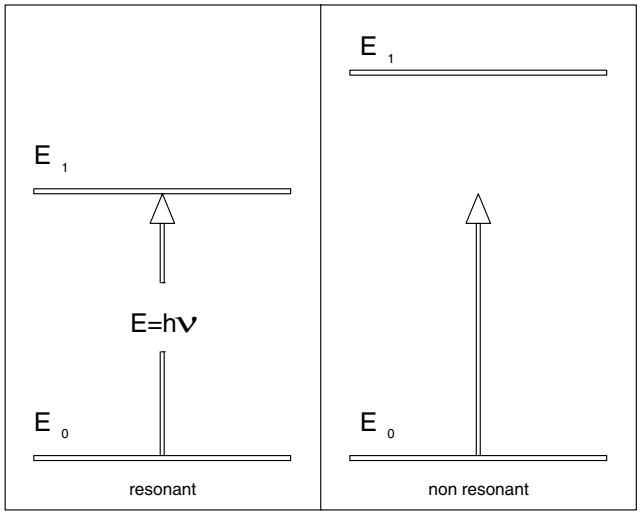



**Fig. 1**: **Incoming light is resonant to a transition of the sample (left) and non-resonant (right) for a material with another transition**

There is still an interaction, in which, however, no transition of the electrons takes place. The interaction occurs through the electromagnetic property of light together with the electromagnetic property of matter.

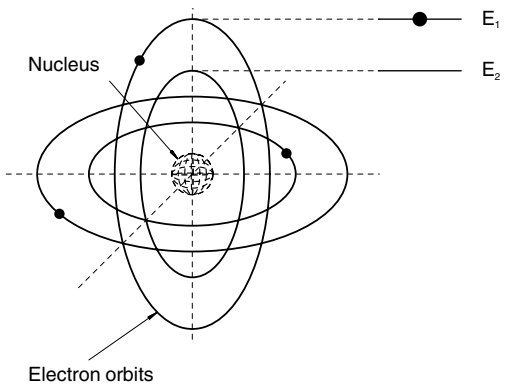



**Fig. 2: Bohr’s Atom**


Dr. W. Luhs **MEOS** GmbH 79427 Eschbach - August 1998/ July 2003

The work and measurements had proved that discrete energy must be anticipated for both resonant cavities and appearances of atomic emissions. Einstein began looking for a single description for both these sources of light. He was able to solve this problem in 1917 when he derived Planck's hypothesis once more in his own way. He thought of a way of combining both light sources. He put an ensemble of "Bohr's" atoms in a resonant cavity at a temperature T. In thermal equilibrium *E(ν,T)* will be an energy distribution which must be formative influenced by the properties of the atoms. Einstein's task was to determine this new energy distribution. In a first step he examined the atom ensemble, which we presume has only two energy levels, as shown in Fig. 9. Since the atoms are exposed to a radiation field, they can take up or absorb energy. The absorption is connected to an emission. If we denote the number of electrons in state 1 as $n_1$, the temporal change of it will be:

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u(\nu) \qquad (1)$$

In this case $u(\nu)$ is the density of energy at the frequency at which the transition from state 1 to state 2 is resonant, i.e. it is the frequency at which $E_2$-$E_1$=$h\nu$ is fulfilled. This frequency is called the resonant frequency. It is evident that the temporal change from $dn_1/dt$ is dependent on the number $n_1$ itself, on one hand, and on the density of energy of the radiation with the frequency $\nu$, on the other.
A constant $B_{12}$ is necessary for a correct equation in terms of dimension. The minus sign is required because the number of electrons in state 1 decreases through the absorption .
The same observation is carried out for state 2. We will call the number of electrons in this state $n_2$. The electrons return to state 1 from state 2 whilst emitting radiation. The transition from 2 to 1 is released (induced) by the existing radiation field of the resonator and takes also place coincidentally (spontaneously). So, two types of emission are responsible for depopulating state 2, the induced and the spontaneous emission. The temporal change in the number $n_2$ is

$$\frac{dn_2}{dt} = -B_{21} \cdot n_2 \cdot u(\nu) - A_{21} \cdot n_2 \qquad (2)$$

Nothing has been left out of the last term since the spontaneous emission does not depend on the surrounding radiation field and is of a statistical nature. It takes place even when there is no radiation field. Until the principles of quantum mechanics were defined by Heisenberg and Schroedinger, it was accepted that spontaneous emission was similar to radioactive decay, in that it could not be influenced from the outside. Quantum electrodynamics has shown that a spontaneous emission is an emission induced by zero point energy. So as not to take this too far at this point, the following must be noted with reference to zero point energy. In the cavity there is an average field energy of at least $E_0$=1/2 $h\nu$. The spontaneous emission is triggered off by this energy. Let us go back to our resonant cavity-two level atom system. In stationary equilibrium, the same number of electrons must go from state 1 to 2 (with a photon being absorbed from the radiation field) and vice-versa (emission of a photon into the radiation field)

$$\frac{dn_1}{dt} = \frac{dn_2}{dt} \qquad (3)$$

or

$$B_{12} \cdot n_1 \cdot u(\nu) = B_{21} \cdot n_2 \cdot u(\nu) + A_{21} \cdot n_2 \qquad (4)$$

The Boltzmann distribution is also valid in the thermal equilibrium for the population numbers of level 1 and level 2

$$n_2 = n_1 \cdot e^{-\frac{E_2 - E_1}{k \cdot T}} \text{ or } n_2 = n_1 \cdot e^{-\frac{h \cdot \nu}{k \cdot T}} \qquad (5)$$

By substituting (5 ) in ( 4 ) we get

$$u(\nu, T) = \frac{A_{21}}{B_{12}} \cdot \frac{1}{e^{-\frac{h \cdot \nu}{k \cdot T}} - B_{21}/B_{12}} \qquad (6)$$

Since Planck's law must be valid also in equilibrium we get by comparison of ( 1.3.33 ) with ( 1.4.6 ) the meaningful Einstein coefficients:

$$B_{12} = B_{21} \text{ und } \frac{A_{21}}{B_{12}} = 8\pi \cdot \frac{h \cdot \nu^3}{c^3} \qquad (7)$$

## 2.2 Natural line width

Let's look on Eq. **( 1 )**:

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u(\nu)$$

$B_{12}$ can be considered as the probability for a transition from level 1 to level 2 by absorption.
This is also analogous to the coefficient $B_{21}$, which however indicates the probability of the reverse process, i.e. the emission.
The coefficient for the spontaneous emission $A_{21}$ gives us another interesting piece of information on the system, which is easy to find.
Let us take, for example, the process of the spontaneous emission by itself.

$$\frac{dn_2}{dt} = -A_{21} \cdot n_2 \qquad (8)$$

This differential equation can be solved using the additional equation

$$n_2(t) = C \cdot e^{-\alpha \cdot t} \qquad (9)$$

$\alpha = A_{21}$ can be found by comparing the two and the solution will then be

$$n_2(t) = n_2(t=0) \cdot e^{-A_{21} \cdot t} \qquad (10)$$

Fig. 3 shows this function graphically. This curve and therewith $A_{21}$ can be determined experimentally. The time $t$ which $n_2(t)$ took to reach the value $n_2\ (t=0)\ e^{-1}$ must be deduced. The result will then be $t = 1/A_{21}$.
Obviously the reciprocal value of the Einstein coefficient $A_{21}$ represents a suitable definition for the "life time $\tau$ of a state".

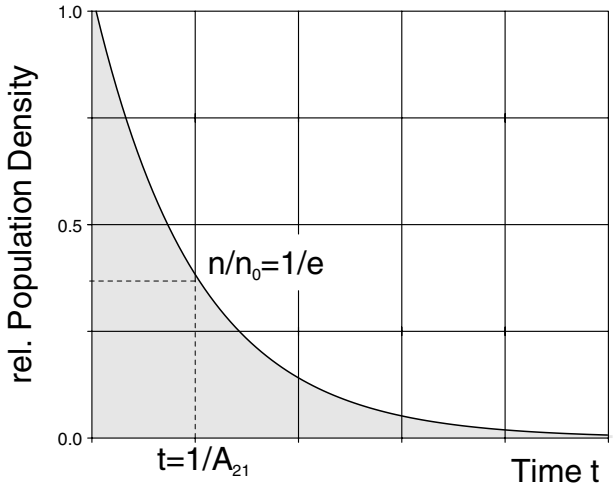



**Fig. 3: Population decay curve of a state**

More information can be obtained from the decay curve. photons or a radiation field are produced because of the transition from state 2 to 1.
However, the intensity of the radiation decreases exponentially with the time (Fig. 4). In view of the preceding findings, the frequency of the radiation should be fixed to $E_2 - E_1 = h\ \nu_0$.

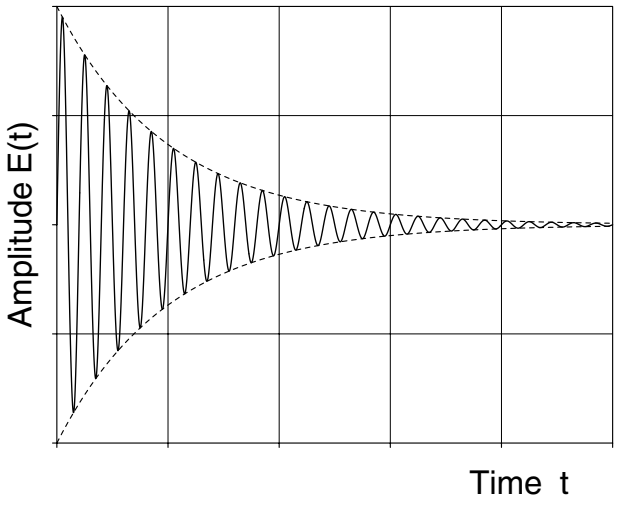



**Fig. 4: Spontaneous emission as a damped oscillation**

A power spectrum of the spontaneous emission is obtained using a Fourier analysis for non-periodic processes, which has the main frequency $\nu_0$ apart from other frequency parts. The result of this kind of Fourier transformation is illustrated in Fig. 5.

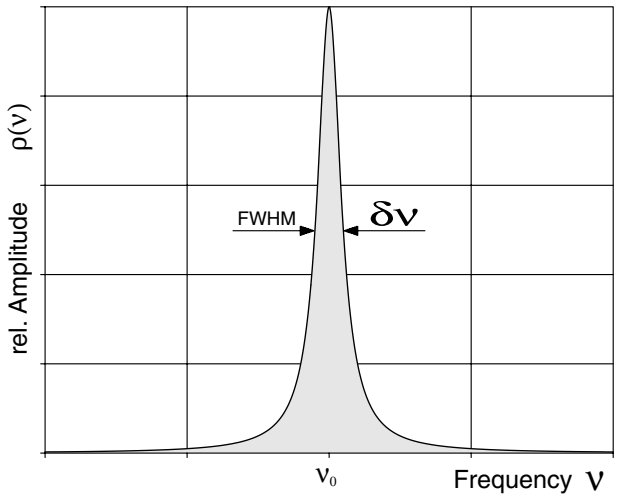



**Fig. 5: The Fourier transformation of a damped oscillation as observed for spontaneous emission. It consists of the transition frequency $\nu_0$ and a complete spectrum described by a Lorentz function.**

The Fourier transformation of the damped oscillation gives the following result:

$$\rho(\nu) = \frac{1}{4\pi^2 \cdot (\nu - \nu_{21})^2 + (1/2 \cdot \tau_s)^2} \qquad (11)$$

This type of curve represents a Lorentz curve. $\nu_{21}$ *(or* $\nu_0$*)* is the resonant frequency and

$$\tau_s = \frac{1}{A_{21}}$$

the average life time of state 2. The FWHM (Full Width at Half Maximum) of the curve as shown in Fig. 5 is calculated by inserting the value of $\rho(\nu) = 1/2$. The result is:

$$\delta(\nu)_{nat} = \frac{1}{2\pi} \cdot A_{21} \qquad (12)$$

which is the natural line width of a transition, defined by the Einstein coefficient $A_{21}$ which has a particular value for every transition. The results obtained can also be interpreted as if the state 2 did not have any clearly defined energy, but a broadening with half-width $\Delta E = 2\ \pi\ h\ A_{21}$. This means that the state is somewhat blurred. Quantum mechanics has shown this effect to be extremely important. It is known as the Heisenberg uncertainty principle, after the person who found it. In the case of normal optical transitions the value of $\tau_s$ lies between $10^{-8}$ to $10^{-9}$ seconds. This life time, determined by spontaneous transitions alone, is crucial for the so called **natural width** of a spectral line. To clarify the ways in which we term things, we must emphasise briefly at this point, that there is a difference between the width of a state and the width of a line, as well as between the terms state and line. There are always states for atoms and it is never stated whether the state is occupied or empty. A line is only formed if an emission is caused by the transition from, for example, state 2 to 1. The line is a word commonly used by spectroscopists. They use their spectroscopes to produce photo-

graphic plates, for example, on which fluorescent light is shown according to its wavelengths. The use of slits in the optical beam path makes it easier to evaluate the spectra. A line spectrum of this kind is shown in Fig. 6.

**Fig. 6: Recording the emission of a light source with corresponding energy levels results in a line spectrum**

Apart from the emission wavelengths, the spectrum in Fig. 6 shows the line widths. It must be observed, in this case, that the measuring apparatus makes the line widths seem wider than they actually are. Naturally, it was the aim of spectroscopists to create instruments which could give the closest reading of the actual line widths.

## 2.3 Homogeneous line broadening

A line is homogeneously broadened when all the atoms or molecules have the same characteristics and all of them interact with their environment in the same way. The natural broadening is a homogeneous broadening, since it is the same for all atoms and molecules in an ensemble. Homogeneous broadening can be found in solids with regular crystal structure in which the atoms considered are in equivalent lattice sites. The interactions with the crystal lattice lead to a broadening of the states that is far beyond the natural width, but which is homogeneous when the lattice sites are symmetrical and of equal value. Gases are known for their inhomogeneous broadening and this will be discussed in the next section. In this case the absorption and emission lines are no more homogeneously but inhomogeneously broadened.

## 2.4 Two level system

Now that we have learnt a few aspects of the interaction of light with matter, in the following example of a simple two level system, the description of absorption and emission with the help of the Einstein's coefficient shall follow. Finally, we will learn the inverse process of Absorption, the amplification of photons.

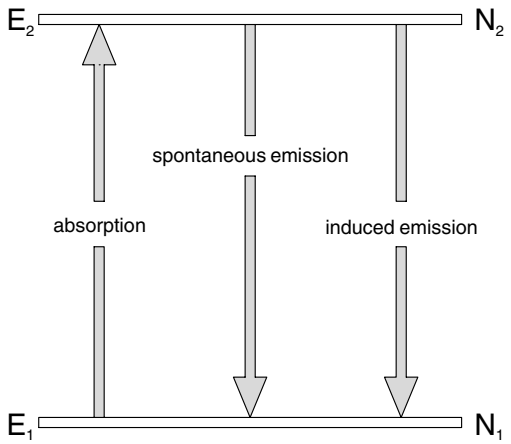



**Fig. 7: Two level system**

We use the rate equation model, a formalism, whereby the temporal change of the population density can be determined for each of the energy levels involved. For this, let us take a number of atoms $N$ or molecules which are presently in the Ground state ($E_1$). We irradiate this sample now with photons with the spectral energy density $\rho(\nu)$ i.e. Energy per Volume element and per Frequency interval $\Delta\nu$. Since this concept is used often today, at this stage we will take a closer look at its definition. A photon possesses the energy $E = h\nu$. When, in the volume $dV$, the number of photons is $N$, the energy density is:

$$\frac{N \cdot h \cdot \nu}{dV}$$

Now, not all the photons have necessarily the same frequency, which is why for further classification we require that only those photons whose frequency lies within the interval $\Delta\nu$ shall be considered. The distribution of the number of photons follows a function which we denote by $f(\nu)$ without, for the present, defining it any further. We arrive consequently at the expression for the spectral energy density:

$$\rho(\upsilon) = \frac{N \cdot f(\nu)}{dV} \cdot \frac{h \cdot \nu}{\Delta\nu}$$

The density of photons $\rho$, its number $N$ per volume element $dV$ is related with the spectral energy density:

$$\rho(\upsilon) = \rho \cdot h \cdot \nu \cdot \frac{f(\nu)}{\Delta\nu} \qquad (13)$$

Under the influence of the photon field the population numbers $N_1$ and $N_2$ would change. Now we consider the temporal changes in the numbers to then determine the stationary state. The population number of the level $E_1$ diminishes in the absorption process:

$$\left.\frac{dN_1}{dt}\right|_{Absorption} = -B_{12} \cdot \rho(\nu) \cdot N_1$$

Through the absorption process the level $E_2$ is populated:

$$\left.\frac{dN_2}{dt}\right|_{Absorption} = B_{12} \cdot \rho(\nu) \cdot N_1 \qquad (14)$$

Through spontaneous and induced Emission the level 2 is, however, also depopulated:

$$\left.\frac{dN_2}{dt}\right|_{spontaneous} = -A_{12} \cdot N_2 \qquad (15)$$

$$\left.\frac{dN_2}{dt}\right|_{induced} = -B_{21}\cdot\rho(\nu)\cdot N_2 \qquad (16)$$

The total change in the population of the energy level $E_2$ is therefore::

$$\frac{dN_2}{dt} = B_{12}\cdot\rho(\nu)\cdot(N_1 - N_2) - A_{12}\cdot N_2 \qquad (17)$$

The temporal change in the population density in the state 2 obviously changes the photon density since every absorption process is bound with the destruction of a photon, and every emission process with the production of a photon. The change in the photon density is therefore equal to the difference in the population numbers of the levels considered:

$$\frac{d\rho}{dt} = \frac{dN_1}{dt} - \frac{dN_2}{dt} \qquad (18)$$

with

$$\frac{d\rho}{dt} = \frac{d\rho}{dx}\cdot\frac{dx}{dt} = c\cdot\frac{d\rho}{dx}$$

and equation ( 13 ) we get

$$\frac{d\rho}{dx} = B_{12}\cdot\frac{f(\nu)}{\Delta\nu}\cdot h\cdot\frac{\nu}{c}\cdot\rho\cdot(N_1 - N_2)$$

we use as the abbreviation

$$\sigma_{12} = B_{12}\cdot\frac{f(\nu)}{\Delta\nu}\cdot h\cdot\frac{\nu}{c}$$

$$\frac{d\rho}{dx} = \sigma_{12}\cdot\rho\cdot(N_1 - N_2)$$

and as a solution of the differential equation we get:

$$I = I_0\cdot e^{-\sigma_{ik}\cdot(N_1 - N_2)\cdot x} \qquad (19)$$

In that we use the identity for the intensity *I* and the photon density $\rho$

$$\rho\cdot c = I$$

Amplification then takes place when

$$g = \sigma_{ik}(N_2 - N_1) = \sigma_{ik}\cdot n > 0 \qquad (20)$$

On comparing the result after **( 19 )** with that of the famous absorption rule in classical optics by Beer, it is determined that the strength of the Absorption does not only present a material constant, but additionally depends on the difference of the population numbers $N_1$-$N_2$ and therefore on the intensity of the photon field. This dependence is determined through the solution of the rate equations for the stationary state.

$$\frac{dN_i}{dt} = 0\,.$$

From $dN_2 / dt = 0$ we get the expression:

$$\frac{N_2}{N_1} = \frac{B_{12}\cdot\rho(\nu)}{B_{12}\cdot\rho(\nu) + A_{12}}$$

Thereafter for a very large (large $\rho(\nu)$) photon density $N_2 / N_1$ goes against 1. This means that in this case the population density of both the levels are equally large and according to ( 19 ) no more absorption takes place, the medium has become transparent under the influence of strong photon fields.

Obviously in this two level system it is not possible to produce a population number $N_2$ larger than $N_1$ since according to equation ( 19 ) instead of absorption, amplification takes place.

## 2.5 Optical Amplifier

The optical amplifiers are characterised by the fact that at their output, the number of photons is larger than at their input. A material that possesses this characteristic, must have a structure of energy levels, in which a population inversion, i.e.

$$N_i > N_f$$

can be produced. In this, $N_i$ is the population density of the excited and $N_f$ the lower lying state.

## 2.6 Three-level system

In the last chapter, we found out, that such a situation cannot be produced in a two level system. Therefore, we shall presently attempt a three-level system (Fig. 8).

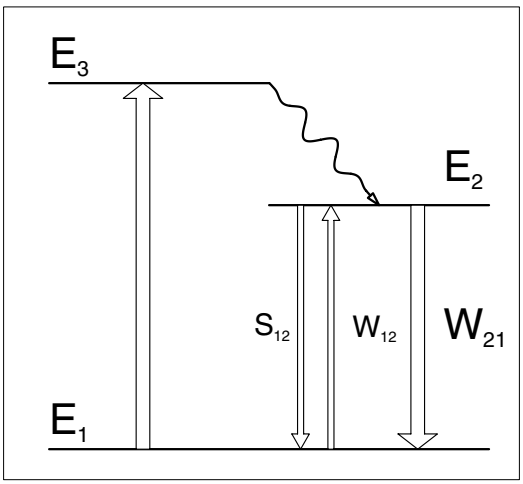


**Fig. 8: Three level system**

There $E_1$ is the initial level, mostly also the ground state of the respective atoms or molecules for the pump process. The atoms or molecules of the ground state are excited through the pump process and populate the state $E_3$ with the population density $N_3$. We proceed on the assumption that the transfer from $E_3$ to $E_2$ occurs very rapidly, so that $N_3 = 0$. The temporal change in the population density of the state $E_2$ results therefore, directly from the decrease in the population density $E_1$:

$$\left.\frac{dN_2}{dt}\right|_{Pumpprocess} = \eta \cdot W_{13} \cdot N_1$$

Here $W_{13}$ is the probability that a particle passes from state 1 to state 3 and $\eta$ for that of the transition from 3 to 2. The product:

$$W_{13} \cdot \eta = W_p$$

represents consequently the pump rate for a particle. The spontaneous Emission affects the state 2 as the second process and leads to a depletion of the state.

$$\left.\frac{dN_2}{dt}\right|_{spontaneous} = -\frac{N2}{\tau_s} = -\Gamma \cdot N_2$$

Here $\tau_s$ stands for the life time of the state 2 and $\Gamma$ as its inverse value. A special feature of the spontaneous emission is that it is not possible to influence it through external fields and depends only on the life time of the levels. In contrast to this, the induced emission depends on the one hand on the existing photon density p and on the difference in population density $N_2$-$N_1$.

$$\left.\frac{dN2}{dt}\right|_{induced} = -\sigma \cdot c \cdot p \cdot (N_2 - N_1)$$

With $\sigma$, the cross section for the induced emission is introduced and $c$ is the velocity of light. It consequently follows that for the total temporal change in population in state 2:

$$\frac{dN_2}{dt} = \sigma \cdot c \cdot p \cdot (N_1 - N_2) - \Gamma \cdot N_2 + W_p \cdot N_1$$

Since each process of state 2 leads to an opposite change in population density of state 1 it is valid for the temporal change of state 1 so that

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt}$$

In every induced process one photon is produced or destroyed. Therefore the photon density $p$ changes respectively:

$$\frac{dp}{dt} = -\sigma \cdot c \cdot p \cdot (N_1 - N_2)$$

photon once produced, however do not stay available for all time. They can be destroyed by other processes and their density decreases with a time constant $\tau_{ph}$. We formulate these losses as loss rate and its temporal change is:

$$\frac{dp}{dt} = -\frac{p}{\tau_{ph}}$$

For the total change in photon density we finally get:

$$\frac{dp}{dt} = p \cdot \left( \sigma \cdot c \cdot (N_2 - N_1) - \frac{1}{\tau_{ph}} \right)$$

We use the following abbreviations:

$$n = N_2 - N_1 \text{ and } n_{tot} = N_1 + N_2$$

so we get:

$$\frac{dn}{dt} = -2\sigma cpn - \Gamma (n_{tot} + n) + W_p \cdot (n_{tot} - n) \quad \textbf{( 21 )}$$

and

$$\frac{dp}{dt} = p \cdot \left( \sigma \cdot c \cdot n - \frac{1}{\tau_{ph}} \right) \quad \textbf{( 22 )}$$

The equations ( 21 ) and ( 22) build a couple of simultaneous differential equations, for which no analytical solutions are known. Merely for a few special cases, solutions are possible. For the stationary state, i.e. the temporal change in population inversion is zero, respectively.

$$\frac{dn}{dt} = 0$$

so from ( 21 )we get

$$n = N_2 - N_1 = \frac{n_{tot} \cdot (W_p - \Gamma)}{W_p + \Gamma + 2\sigma \cdot c \cdot p} \quad \textbf{( 23 )}$$

We use eq. ( 20 ) for the amplification, and get ( 23 )

$$g = \frac{\sigma \cdot n_{tot} \cdot (W_p - \Gamma)}{W_p + \Gamma + 2\sigma \cdot c \cdot p}$$

Obviously amplification occurs only when the pump rate is larger than the rate of spontaneous emission. This is achievable when the life time $\tau_s$ is very large. Especially

suitable are therefore metastable states. We further discover that with increasing photon density the amplification decreases.

Thus, we have been able to show that in a three level system, optical amplification can be achieved.

It shall not pass without mention that four level system show more favourable characteristics in a few respects.

Still the level system of EDFA is a three level system which shall be illustrated in the following. Now the question arises, why this Erbium doped Fibre was chosen as a candidate for this application. The main reason lies therein, that this material shows suitable transition at 1550 nm. This wavelength is of extraordinary importance for the communication technology with glass fibres. This wavelength falls in the so-called second absorption window. Now it is not sufficient just to have a suitable transition with this wavelength, but one should be able to excite it with simple means. The first step of the experiment therefore contains the analysis of the Absorption- and Fluorescence spectrum.

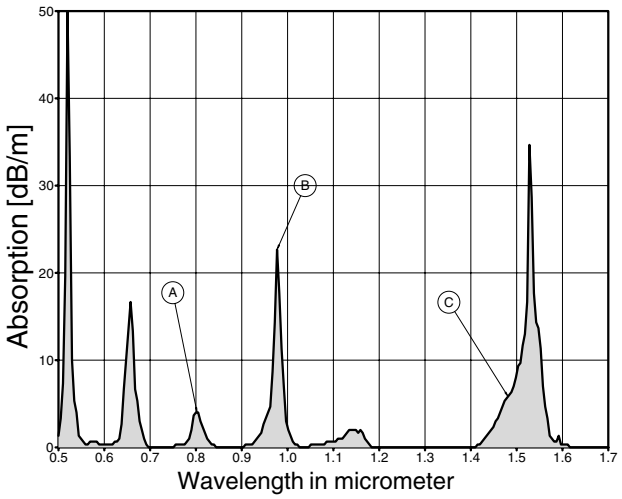



**Fig. 9: Absorption spectrum of an Erbium doped fibre, A Pump band for 800 nm, B 980 nm and C 1480 nm**

The areas marked in the Fig. 9 can be pumped with currently available Laser diodes. It has however turned out that the use of transition B is connected with many advantages. A detailed description of the characteristics of all the possible pump configurations is to be found in: „Optical Fiber Amplifiers: Design and System Application“ by Anders Bjarklev published in Artech House Boston London, ISBN 0-89006-659-0.

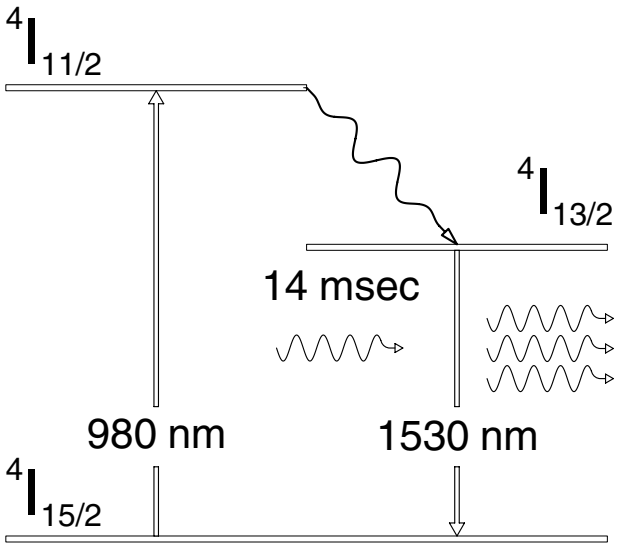



**Fig. 10: Three level system of EDFA**

In the Fig. 10 we recognise the well known level system from the preceding chapter. The pump transition occurs between the states ${}^4I_{15/2} \rightarrow {}^4I_{11/2}$, followed by a quick transfer of the ${}^4I_{11/2} \rightarrow {}^4I_{31/2}$ and finally as radiative transition back to the ground state ${}^4I_{15/2}$. With a comparatively extremely long life time of 14 msec, this system fulfils the requirements for the production of the desired population inversion.