Better Technology. Better Metrology.™

OEM

A Primer on Displacement Measuring Interferometers

A Displacement Measuring Interferometer (DMI) measures linear and angular displacements with very high accuracy and precision. DMI's are used in a variety of applications which can be broken into two broad categories:

- high resolution real time position control systems, such as those used in semiconductor lithography, e-beam and laser reticle writers, CD measurement tools, process equipment, and memory repair tools,
- characterization of high resolution, high frequency mechanical motions such as piezo transducers, linear and rotary scale calibration, AFM stage calibration.

This primer outlines some of the practical issues which face the user when integrating a DMI as part of an instrument design or as a laboratory tool.

zygo

Revised 1/99

Outline

- DMI introduction
- Homodyne v.s. Heterodyne
- System components & configurations
- System error sources
- DMI Applications
- Application specific interferometers

The document begins with a review of optical concepts including interferometry, system resolution, measurement limits and basic subsystems of the displacement measuring interferometer.

An electronics overview follows with a comparison of Heterodyne and Homodyne detection schemes.

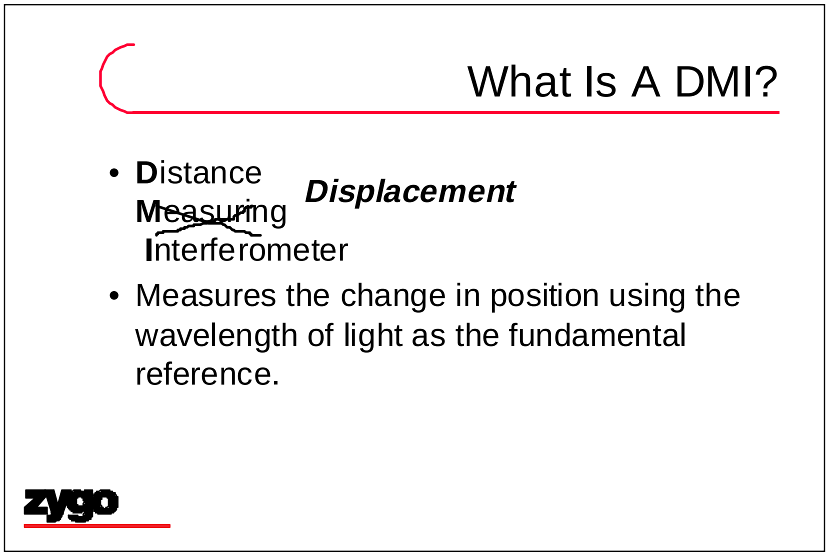
Discussion of the instrumentation concentrates on optical configurations and some common system components.

DMI system design reviews error sources and emphasizes how the user can minimize the effect on the system accuracy by stepping through an error analysis.

Some typical static and dynamic applications are reviewed followed by an overview of some application specific interferometers.

Section 1

- DMI Introduction
- What is a DMI?
- Why a DMI?
- Optical Concepts
  - » Optical Phase
  - » Interference
  - » Polarization



A DMI uses the physical phenomenon of interference of light to measure displacement, i.e., how far something moves. The measurement is relative, not absolute, so the more common name of “distance measuring interferometer” is a misnomer. DMI’s offer the most accurate and sensitive method of tracking linear motion over ranges from fractions of a nanometer to meters.

Why a DMI?

- Geometric Accuracy
- High resolution (up to 0.15 nanometers)
- Long range capability (> 10 meters)
- Measure multiple degrees of freedom simultaneously
- High measurement bandwidth
  (Velocities up to 4.2 meters/second)

zygo

Due to their inherent accuracy, DMI's have become an attractive tool for the most demanding displacement measuring applications. The fundamental accuracy of a DMI system is based on the precise knowledge of the wavelength of light.

A DMI allows the user to minimize geometrical errors, such as Abbe offset error and opposite axis error, that are associated with mechanical displacement measuring techniques.

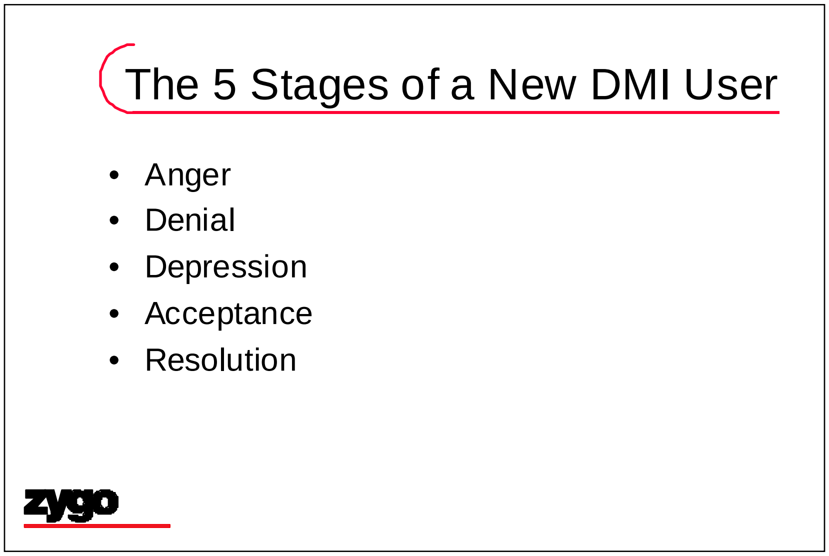
Depending on the system configuration, a DMI can resolve displacements to 0.15 nanometers and track velocities up to 4.2 meters per second over a large displacement range.

DMI's allow the user to measure displacements at the point of interest.   With the simple detection scheme of a heterodyne system, multiple axes can be measured simultaneously (X, Y, θ & more).

Most DMI systems are easy to use and align. Using interferometers with application specific designs, the measured optical path change can be related to physical quantities such as linear displacement, angular displacement, straightness of travel, flatness, squareness, and parallelism, as well as changes in the refractive index of air.

Why a DMI?

- Noncontact
- Allows measurement at the point of interest (on the actual axis of motion)
- Easy to use and align



Thanks to the accuracy and resolution of a DMI system, when one is implemented into a new application the user is often faced with unexpected and often undesired results. This can send the user through a progression of states that typically ends with the acceptance of the data and a potential challenge to resolve undesired system error sources.

DMI Chronology

Resolution	Year	Description
$\lambda/8$	1965	DC Commercial Systems
$\lambda/16$	1970	AC Commercial Systems
$\lambda/512$*	1987	20 MHz Heterodyne
$\lambda/2048$*	1996	State of the Art Electronics

...and it just keeps getting better

* represents linear resolution with two pass interferometer

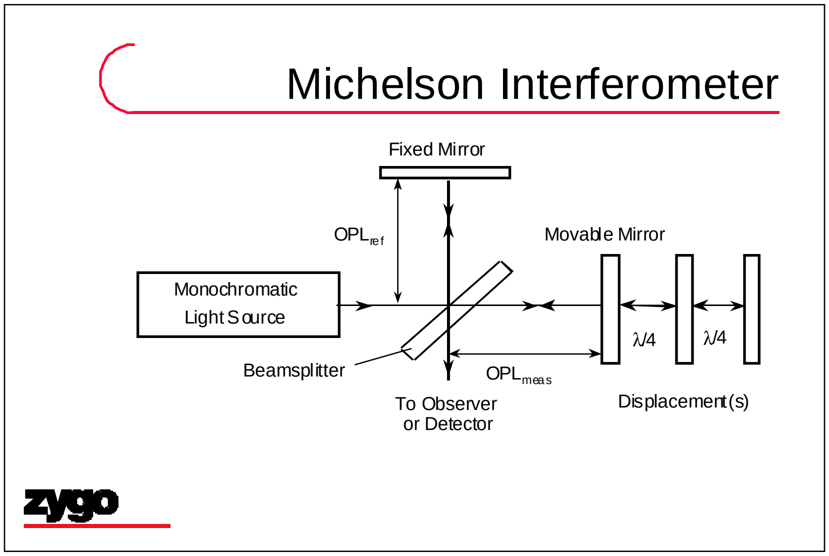
The concepts of fringe measurement and interferometry in general were first heavily experimented on by Albert Michelson in the late 1800’s in a series of classic experiments discussed in most introductory Physics texts. Michelson was certainly able to measure to less than a half wave.

As with many discoveries, the necessary electronics and other associated technologies needed to be developed to fully exploit basic interferometric principles.

Optical Concepts

- Waves
  - » Wavelength/frequency
  - » Interference
- Polarization
  - » Types
  - » Modification

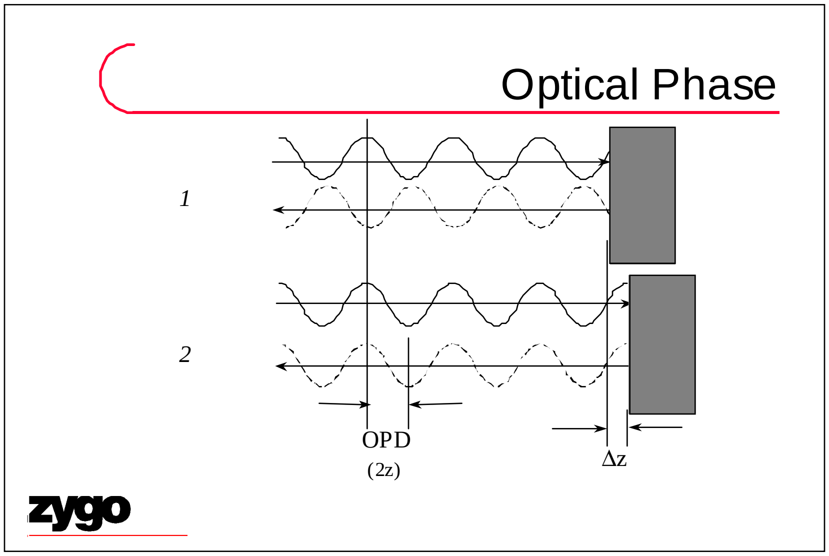
zygo



This is the basic Michelson interferometer. Monochromatic light is directed at a half-silvered mirror that acts as a beam splitter. The beam splitter transmits half the beam to a movable mirror and reflects the remainder at 90 degrees to a fixed mirror. The reflections from the movable and fixed mirrors are recombined at the beam splitter where their interference is observed. With the mirrors exactly aligned and motionless, so that the recombined beams are parallel, an observer will see a constant intensity of light.

When one of the mirrors is displaced in a direction parallel to the incident beam the observer will see the intensity of the recombined beams increasing and decreasing as the light waves from the two paths constructively and destructively interfere. A cycle of intensity change of the interference of the recombined beams represents a half wavelength displacement of movable mirror travel (because the path of light corresponds to two times the displacement of the movable mirror). If the wavelength of the light is known the displacement of the movable mirror can be accurately determined.

An important characteristic of interferometry is that only the displacement is measured, not the absolute position. Therefore; the initial distance to the movable mirror is not measured, only the change in position of the mirrors with respect to each other can be determined.



All interferometers define an interference cavity in which two beams are created; measurement and reference. Typically, the beams are split and follow separate optical paths. When the beams recombine, they are shifted in phase relative to each other.

In the top figure (1) an incoming light ray is shown reflecting from a target mirror. The wave cycle of the light is superimposed on the ray. The optical path length (OPL) of the ray can be thought of as being some number of wavelengths long.

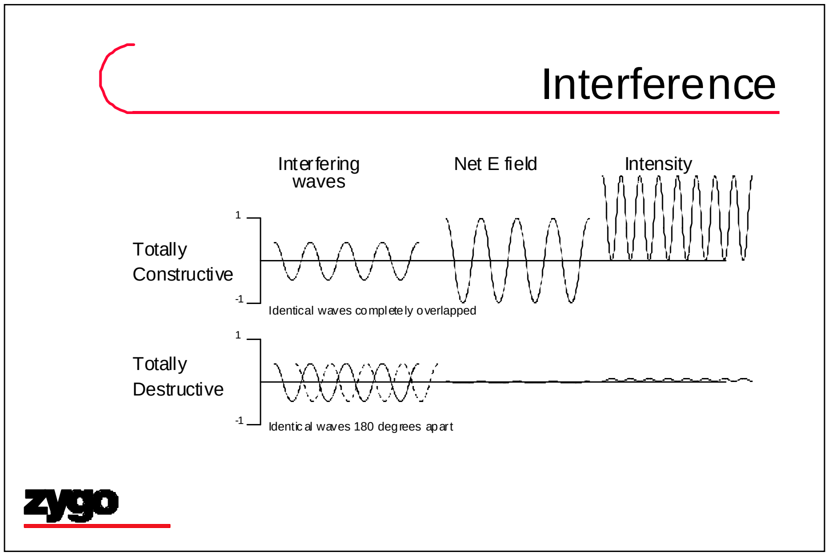
In the bottom figure (2), the target mirror has been moved, therefore OPL increases. From a standpoint of wave cycles the return ray is delayed in the cycle; this is referred to as a phase delay. The optical path difference (OPD) of the two return rays is indicated and corresponds to the motion of the target mirror.

The OPD is equal to the difference in OPL at the time the two individual measurements were acquired;

$$OPD = OPL1 - OPL2.$$

In this configuration the OPD equals twice the target mirror displacement;

$$OPD = 2z.$$



When two waves interfere, the independent electric (E) fields sum to generate a net E field. Intensity (I) is equal to the square net E field.

I = E2

If waves intersect in phase, totally constructive interference occurs. This is the condition of maximum brightness. When the waves are 180° out of phase, totally destructive interference occurs. This is the condition of complete extinction.

When the target is displaced by half of the wavelength, the intensity goes through exactly one period. This corresponds to a light-dark-light transition.

What is a Fringe?

A change in optical phase.

Fringe = 1 full cycle of light intensity variation
(light to dark to light) or (360 degrees of phase)

Scale Factor = (# fringes for displacement of λ) -1

Displacement = Fringes * Scale Factor

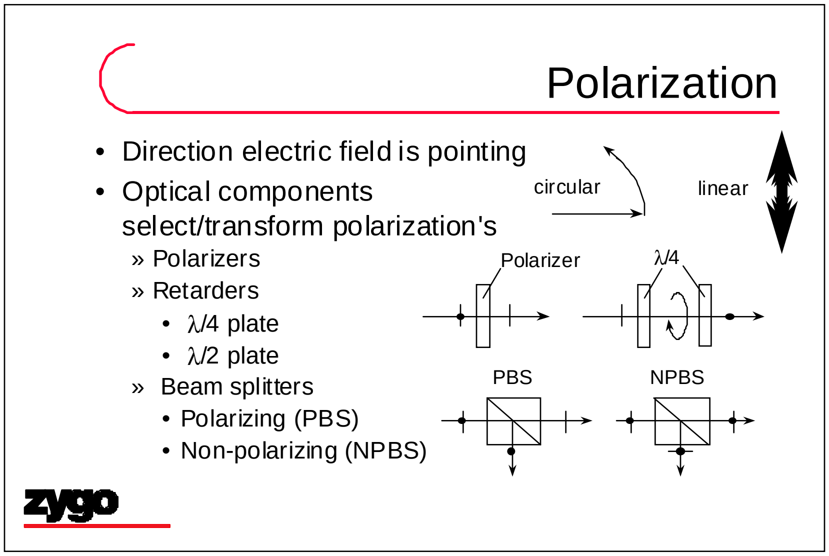
zygo

Early use of interferometers for displacement measurement were based on the counting of fringes. This limited the resolution to 1 fringe and the users ability to interpolate partial fringes.

A Homodyne DMI uses the electronic equivalent of the fringe counting principle, and intensity changes.

A Heterodyne DMI uses electronics to measure changes in optical phase. Note there is another significant point in that a Heterodyne DMI will employ a measurement and reference beam of different frequencies. Going back to the Michelson example, one beam, of frequency 1 will be the reference, the second beam (overlapping the first) of frequency 2 will be the measurement.

The phase change is related to the actual displacement through a scale factor. This scale factor is determined by the optical configuration of the interferometer. In the case of the Michelson geometry, the scale factor is 1/2. This is because any change in the target mirror position changes the optical path length of the measurement beam by twice this amount.



For two beams of light to interfere, the beams must have the same polarization state. In a DMI system this is achieved through the use of a mixture of polarizing and non-polarizing optical components.

Only a single polarization state can transmit through a polarizer. The orientation of the transmitted polarization state is based on the angle of the polarizer in the optical path.

Waveplates or retarders change the polarization state of the light. A quarter waveplate converts linearly polarized light to a circular polarization state. A half waveplate will rotate the plane of polarization e.g. from horizontal to vertical.

Polarizing and non-polarizing beam splitters are used in DMI applications. The non-polarizing beam splitters are used to split portions of the source beam to accommodate multiple axes of measurement. Polarization beam splitters are an integral part of the interferometer. A polarization beam splitter separates the source into the measurement and reference legs.

Section 2

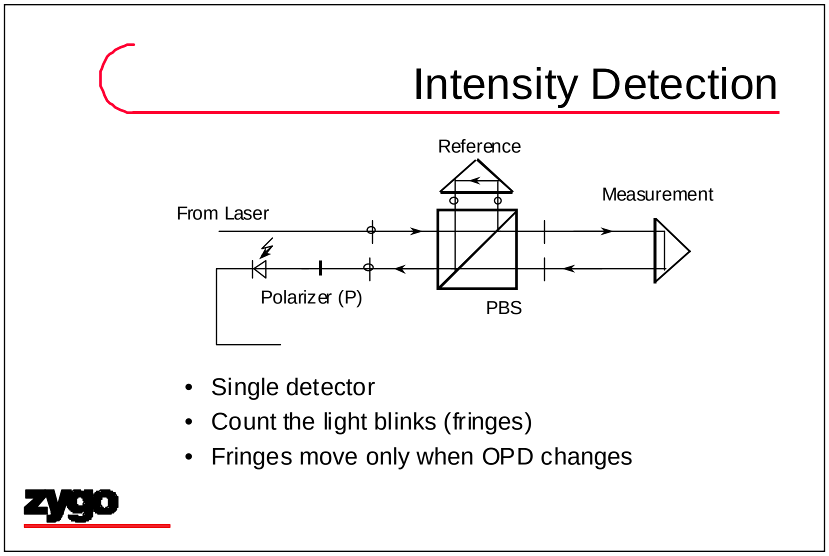
- Homodyne v.s. Heterodyne
- Homodyne (Single Frequency or DC)
  - » Intensity Detection
- Heterodyne (Two Frequency or AC)
  - » Phase Detection
- Detection Bandwidth
- Error Detection Schemes

zygo

A simple homodyne detection scheme is reviewed along with two variations that allow for direction sensing and power normalization.

Heterodyne detection and two techniques used to generate a heterodyne source are described.

A comparison of the detection bandwidth and error detection capabilities of heterodyne and homodyne systems is followed by a summary of the features of the two displacement measurement technologies.



Homodyne = single frequency = DC

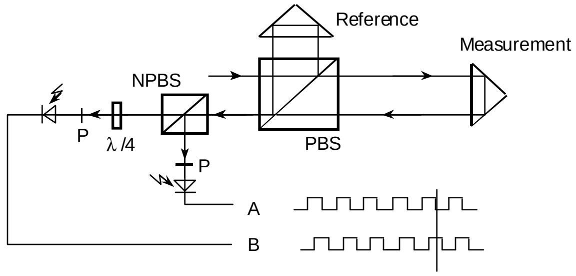
A homodyne interferometer system is made up of a laser source, polarization optics, photodetector(s) and measurement electronics. Depending on the features the user requires from a homodyne system (direction sensing, power normalization, etc), the detector configuration can become complex.

A homodyne source is typically a HeNe laser that outputs a single frequency beam consisting of two opposing circularly polarized components. The beam is split into the reference and measurement legs of the interferometer by a polarization beam splitter (PBS). Following a reflection off their respective targets, the beams return back to the beam splitter.

In order to observe interference, the two beams must have the same polarization. This is accomplished using a linear polarizer oriented at 45° to the two polarization's prior to the photodetector. The signal is run through a Schmidt trigger or similar electronics to locate the zero crossings. Counting the zero crossings is equivalent to counting every half fringe.

Some limitations to a simple homodyne system shown include the inability to detect the direction sense of the target and sensitivity to changes in the beam power and system alignment.

Quadrature Detection

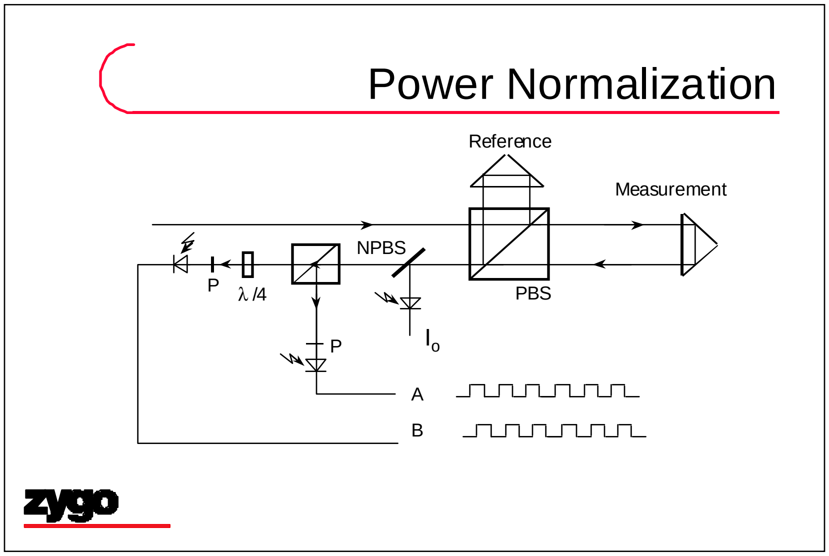


Relationship of A & B edges yields direction sense

To achieve direction sensing capability from a simple homodyne system, the output signal is split by a non-polarizing beam splitter (NPBS) and a second detector is added. Half of the signal is unchanged (A) and the other half (B) is retarded by a quarter waveplate. The waveplate changes the relative phase of the two linearly polarized beams by 90° (1/4 fringe).

Quadrature signals provide direction sensing capability by monitoring the direction of the rising and falling edges of each of the square waves. Since homodyne systems measure intensity changes, the detection electronics require the system to be moving to be able to obtain a displacement signal.

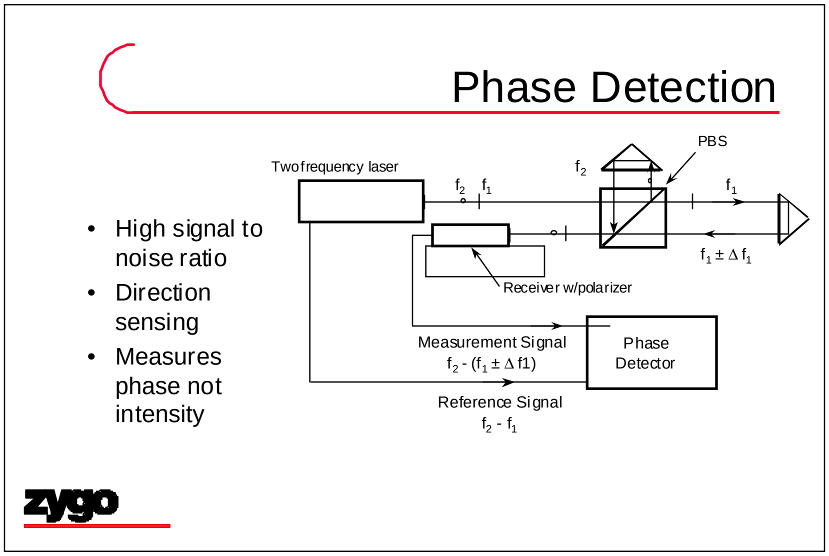
The system described above remains sensitive to intensity variations of the laser and variations in the responses of the photodetectors. A change in source intensity or detector response will be mistakenly recorded as displacement.



To minimize errors caused by fluctuations in laser intensity detectors can be added. The example depicts a homodyne system with quadrature output and an additional detector for normalizing the laser intensity (Io). Some homodyne systems will use more than one detector. For this scheme to work it is necessary to have a good signal to noise ratio at each of the detectors.

Since Homodyne systems detect changes in intensity, the following conditions will can also cause errors:

- beam intensity profile changes during displacement,
- measurement & reference beam overlap changes during motion,
- non-ideal characteristics of the photodiodes.

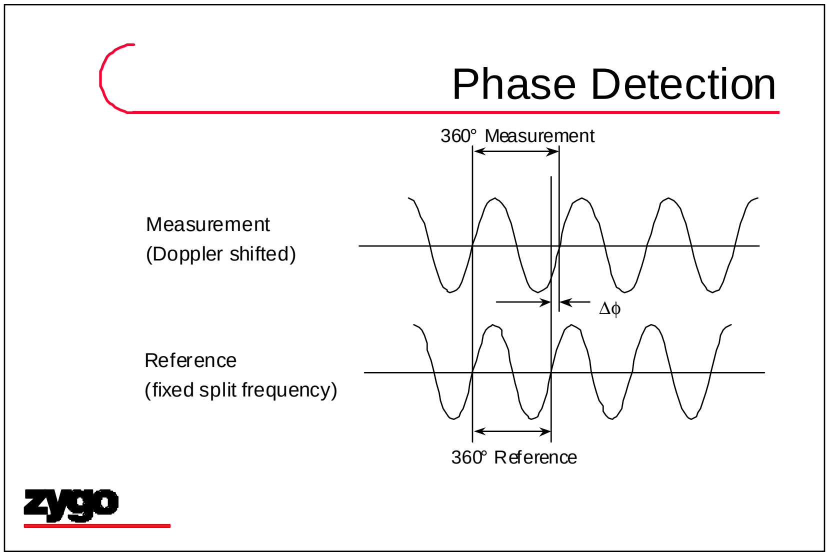


Heterodyne = two frequency = AC

The source for a Heterodyne interferometer system is a highly stabilized, two frequency HeNe laser whose output beam contains two frequency components, each with a unique linear polarization.

In a typical Heterodyne system the laser beam is split into a reference and measurement leg at a polarization beam splitter (PBS). In the single pass interferometer example shown above, one of the frequency components (f_1) is used as the measurement beam and reflects from the moving target back to the beam splitter. The other frequency component (f_2) reflects from a fixed target back to the beam splitter. At the beam splitter, the measurement and reference beams recombine. The recombined beams pass through a polarizer, then a optical interference signal can be monitored at the receiver.

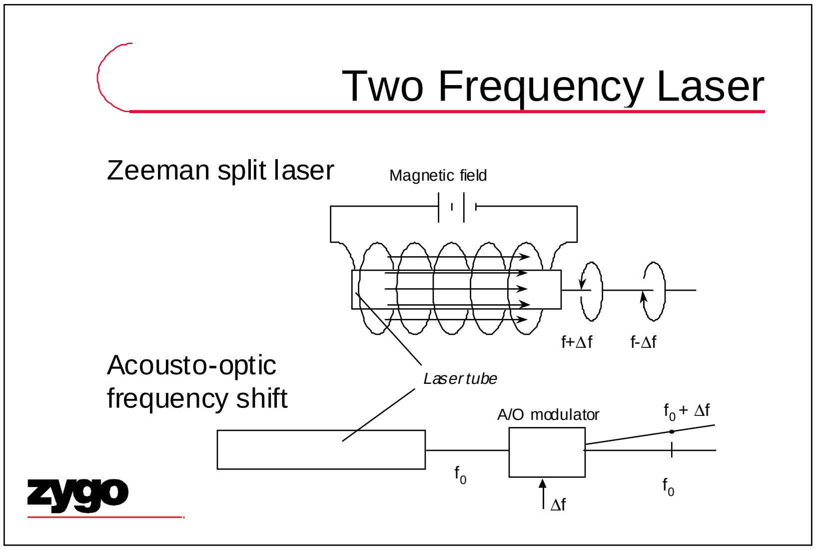
If the movable target remains stationary, the frequency of the optical interference signal (the beat frequency) will be the exact difference between the lasers two frequencies (f_1 - f_2). When the target moves, the frequency of the optical interference signal will be shifted up or down by the Doppler effect ($f_1 \pm \Delta f_1$), depending on the direction of target motion.



Heterodyne detection makes a phase comparison between a measurement signal of unknown frequency to a reference signal of known frequency at discrete time intervals. The zero crossings of the reference signal (in this case the positive zero crossings) are used to indicate the phase of the measurement signal. The change of measurement phase from one reference cycle to another indicates a measurable shift in frequency.

The phase change represents the Doppler shifted frequency that results with movement of the target optic. This shift is monitored by a photodetector and converted to an electrical signal, with the frequency $f = f_2 - (f_1 \pm \Delta f_1)$, where Δf_1 is the Doppler shift. The phase difference between the two signals is measured every cycle and any phase changes are digitally accumulated.

For accurate measurement it is important the phase interpolation scheme (manufacturer specific) be linear to within the required tolerances of the application.

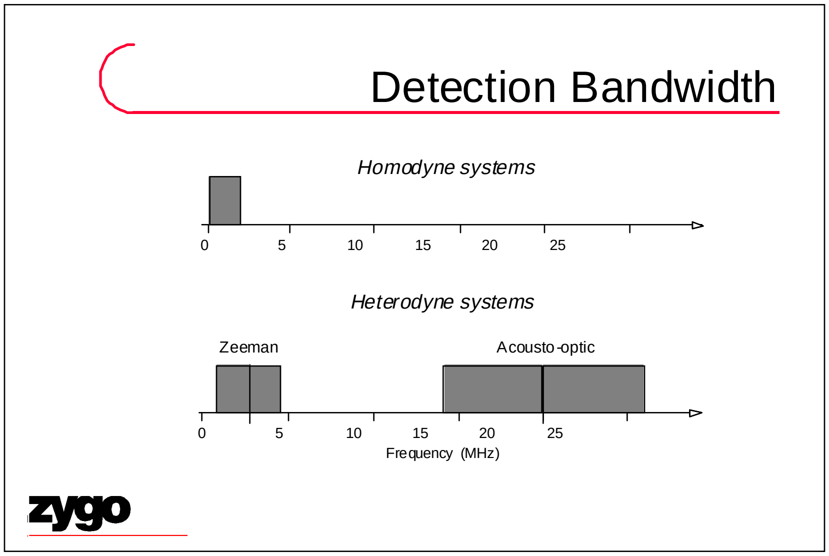


For a DMI system to operate in optical heterodyne mode, the beam from the laser head must have two components that are orthogonally linearly polarized and differ in frequency by a fixed amount. The frequencies must be known and need to remain stable over the lifetime of the laser. Two different methods of generating the frequency split are used in industry; Zeeman technology and an accousto optic method.

The Zeeman technique produces two frequencies by applying an axial magnetic field to the laser tube. The resultant output from the laser consists of a dual frequency beam whose frequency states are circularly polarized in opposite directions. Limitations of the Zeeman effect are :

- a limited difference frequency (maximum of ≈4MHz),
- variation in the split frequency from one laser to the next
- limited laser output power.

The accousto-optic method uses a frequency shifter, such as a Bragg cell, to produce the frequency difference. This technique yields a frequency split that is much greater than that of the Zeeman technique (20MHz). The split also remains constant because the Bragg cell is driven by a stable quartz oscillator.



Detection bandwidth is directly related to the maximum measurable slew rate.

The prime advantage of the two-frequency system is that the displacement information is carried on AC waveforms, or carriers, rather than in DC form. Since AC circuits are insensitive to changes in DC levels, a change in beam intensity cannot be misinterpreted as motion. The AC system achieves greater measurement stability and far less sensitivity to noise (air turbulence, electrical noise, and light noise). Since motion detection information is embedded in the frequency of the measurement signal, only one photodetector per measurement axis is required; thereby decreasing the sensitivity of optical alignment and to the detector gain and bias characteristics.

The homodyne detection must have good signal to noise ratios all the way to zero frequency. This is difficult to achieve since the number of noise sources increases as frequency decreases, e.g., laser power fluctuations.

Among heterodyne systems, it is useful to assess the ratio of the bandwidth to the carrier frequency. The smaller the ratio, the simpler the design of electronics to cover the full range of slew rates. In this respect, the advantage goes to systems with a high carrier frequency.

Error Detection

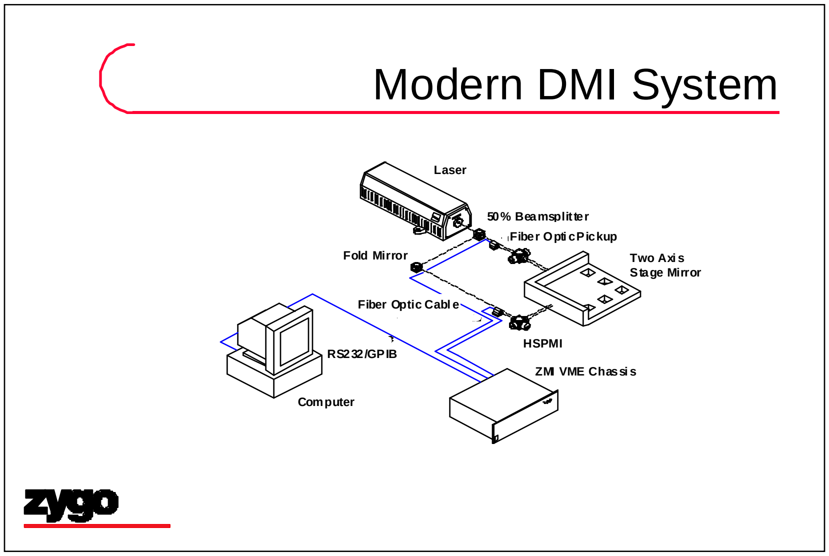
- Error types:
  - » Loss of beam
  - » Exceeding velocity limit
- Homodyne (DC)
  - » Intensity variation
  - » Frequency exceeds detection response
  - » Ambiguous
- Heterodyne (AC)
  - » No frequency
  - » > 2 reference cycles per measurement cycle
  - » > 2 measurement cycles per reference cycle

Homodyne vs. Heterodyne

	Homodyne	*Heterodyne*
Always measuring	no	yes
Direction sensing	quadrature	always
Quadrature output	available	available
Error detection	ambiguous	unambiguous
Intensity sensitivity	yes	no
Sensitive to ambient light	yes	no
Bandwidth of electronics	0 - $2v/\lambda$	f_1-$f_2 \pm 2v/\lambda$
SNR at detector	6-12+ bits	2-3 bits
Multi-axis	limited	yes
Complexity of Receiver(s)	complex	simple
Easy to Align	no	yes

Section 3

- System Components & Configurations
  - » Modern DMI System
  - » Laser Head
  - » Measurement Board
  - » Optical Components
  - » Detectors
  - » Fiber Optics



The modern DMI system consists of a frequency stabilized laser, interferometer optics and measurement electronics. Some recent milestones in DMI technology include:

Synchronized laser heads

The output power of a frequency stabilized HeNe laser is typically less than 700µW. This limits a heterodyne system to about six axes of measurement. Multiple lasers operating at the exact same frequency allow for an unlimited number of measurement axes.

State of the Art Electronics

Linear resolution of 0.15 nanometer (0.0059 µinch) with a 4-pass interferometer and angular resolution of 0.005 arc seconds can be achieved with the latest electronics packages. Velocities of up to 4 meters per second can be tracked with a 0.6 nanometer linear resolution using a linear interferometer.

Fiber Optic Signal Transfer

The latest DMI electronics packages have the detection electronics on the measurement board. This allows for fiber optic transfer of the measurement and reference signals. Due to polarization constraints the laser output cannot be fed to the interferometer without significant efficiency loss and potential polarization mixing errors.

Laser Head

- Stabilized laser
  - » Number of wavelengths for a given distance must be constant
  - » Apparent motion when none occurs
  - » Typically 10-7 - 10-8
  - » Disturbed by laser feedback
- Polarized output
  - » 45° linear for DC systems
  - » 0° and 90° linear for AC systems

zygo

Helium Neon (HeNe) lasers are the most common frequency stabilized source with a typical wavelength around 633 nanometers. The frequency of a DMI laser must be highly stable. If the laser frequency drifts the unequal path length of the interferometer changes its length and the system detects what it believes to be motion of the target.

The primary mechanism for drift of the frequency is change of the laser tube length due to temperature fluctuations. One method of laser stabilization is to wrap a heating coil around the laser tube and then monitor and stabilize the laser output using this coil.

Below, the error in nanometers is shown for different ranges of laser stabilization over various unequal path lengths between the reference and measurement beams of the interferometer.

Path Length	Laser Stabilization 1 ppm	0.1 ppm	0.01 ppm	1 ppb
1 mm	1	0.1	0.01	0.001
1 cm	10	1	0.1	0.01
10 cm	100	10	1	0.1
1 m	1000	100	10	1
10 m	10000	1000	100	10

Measurement Board

- Operation
  - » Detects phase change
  - » Resolution extension of up to l/512
  - » Data output via RS-232, GPIB, ISAbus, VMEbus or P2bus
  - » Real time data output rates up to 10 MHz
- VME & PC based electronics
  - » 6U VME board is most common
  - » ISA based boards available for single axis applications

zygo

The function of the measurement board in a Heterodyne DMI system is to convert a measurement signal from an interferometer and a reference signal from the laser head into measurement data.

The data can be read in the form of an 8, 16 or 32 bit 2's complement word, quadrature or up/down pulses. Direct reads off the P2 connector allow for the fastest real time data acquisition rate; up to 10MHz. The VME bus will output data at a maximum rate of about 3MHz. Quadrature or up/down pulse is the data format that is the easiest to directly integrate to a servo control board for closed loop applications.

Optical Components

- Optics split polarized light into measurement and reference paths.
- Optical configurations provide measurement capabilities differing in:
  - » Degrees of freedom
  - » Scale factor/resolution
  - » Environmental sensitivity
  - » Alignment robustness
  - » Light power efficiency
- Requires a "cooperative" target

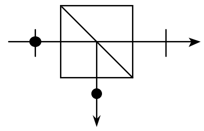
An interferometer consists of a number of optical elements, such as beamsplitters, mirrors, retroreflectors, and waveplates, that are arranged so that the reference and measurement beams travel different optical paths. The components used to make-up the interferometer will determine its resolution, efficiency and thermal stability.

All interferometers are susceptible to path length errors due to thermal and mechanical effects. These effects can be minimized by designing the interferometer so the reference and measurement beams travel equal optical paths through each optical element in the main interferometer body.

In the following pages some basic interferometer components and configurations are shown for heterodyne systems. More complex systems will be discussed later in the document.

Polarization Beamsplitter

- Heart of the interferometer
- Splits source beam into reference and measurement legs.
- Polarization sensitive
- Typically 1/2 or 1 inch cubes

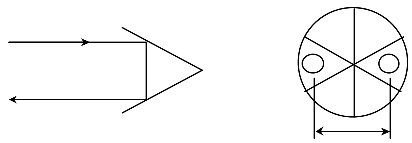


A polarization beamsplitter is at the heart of every interferometer design. The polarization beamsplitter separates the reference and measurement beams (which are at different frequencies).

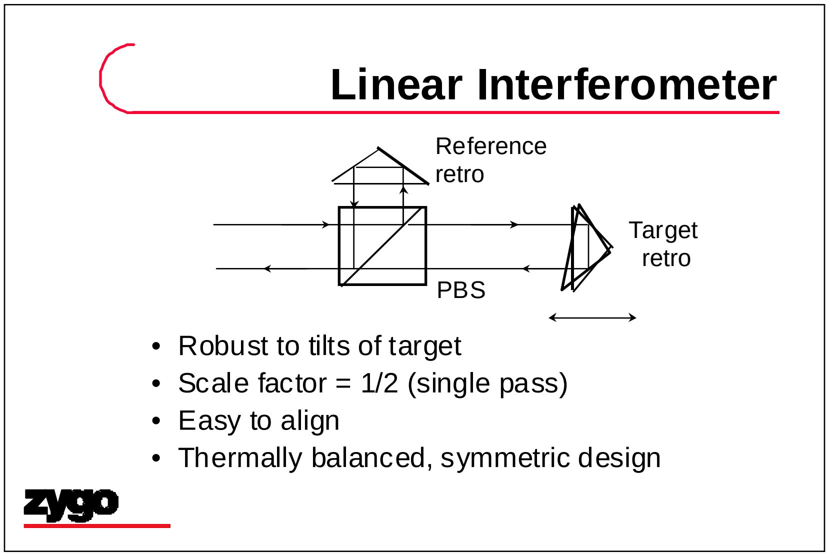
Rotation of the beamsplitter about the optical axis will cause mixing of the polarization states resulting in a measurement anomaly.

Retroreflector

- Output beam is parallel to the input beam, independent of tilt of retro
- Output beam displaced from input by a distance equal to twice the distance the input is located from the retro apex



A retroreflector is used as a target for single pass interferometer configurations. It is also an integral part of other interferometer designs that require multiple passes of the measurement and reference beams.



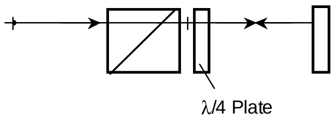
The linear interferometer is a single pass design with a scale factor of 1/2. The interferometer consists of a polarizing beamsplitter cube and two retroreflectors.

A DMI has a limited range of motion. This is dictated by the instrument electronics, the coherence length of the laser head and the degradation of the laser beam profile as it propagates over long distances. Since the measurement beam that exits the linear interferometer makes only one pass to the target this single pass design allows for the maximum range of travel (a two pass design will be limited to half the range of a single pass).

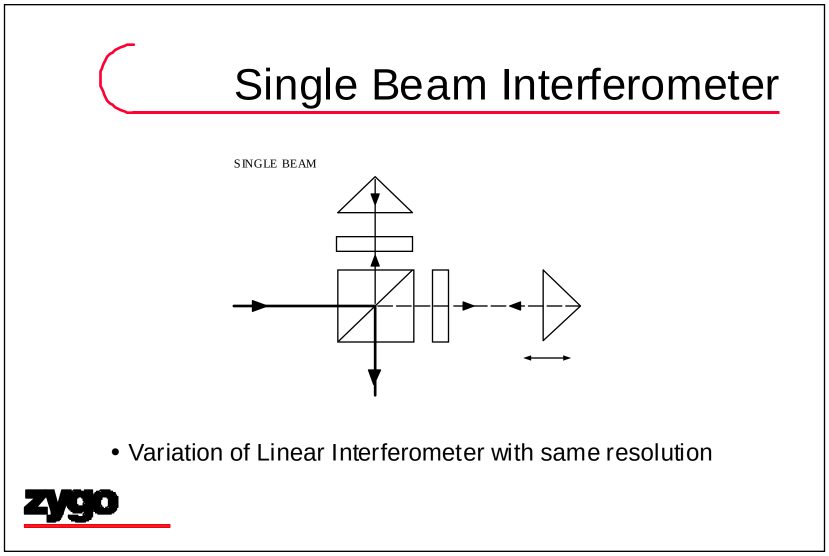
The speed at which the target may be moved is governed by the frequency split of the laser source and the bandwidth of the DMI electronics. The maximum achievable target velocity with a commercially available DMI is 4.2 meters/second.

Quarter Waveplate

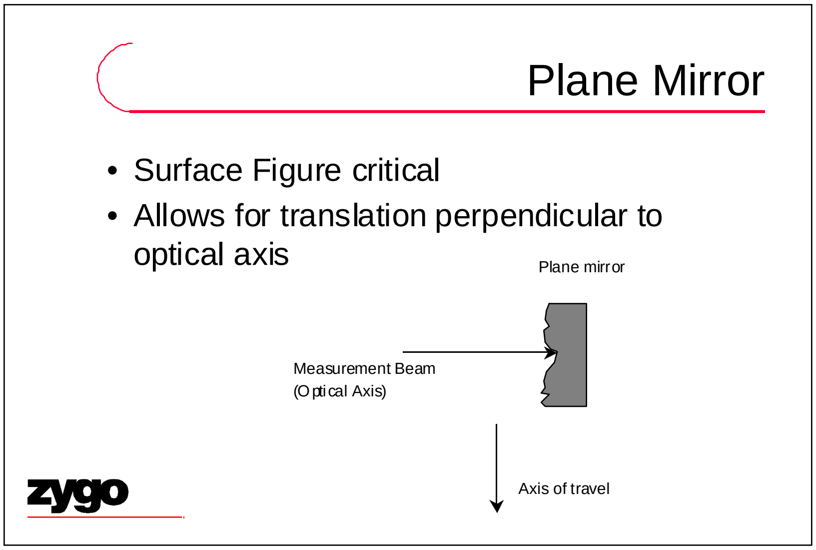
Changes the polarization state of a linearly polarized beam to circular.



A quarter waveplate converts linear polarization into circular. Quarter waveplates are used at the output of all PMI's. Two passes through a quarter waveplate result in a 90° rotation of the beam's polarization state and allows for the measurement beam to make a second pass to the target for increased resolution.



The single beam interferometer is a variation of the linear interferometer that allows the beam to enter the center of the polarization beamsplitter and reflect off the apex of the retroreflector. If the source beam is too small (3mm diameter or less) this design may result in large efficiency losses at the retroreflector unless knife edge retroreflectors are utilized.

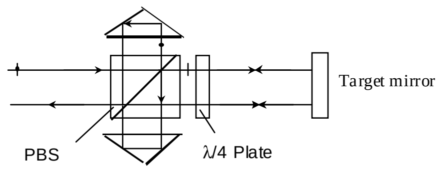


If a target mirror is translating in a direction parallel to the reflective surface, the flatness (surface figure) of the mirror becomes critical. Translation across a deformed mirror will result in the electronics outputting a value that can be read as an apparent displacement of the target.

A target mirror is allowed only minimal tilt during a measurement because as the target tilts, the measurement beam becomes displaced from the reference beam at the receiver.

Plane Mirror Interferometer

- Allows translation of target perpendicular to beam
- Sensitive to tilts of target
- Scale factor = 1/4 (two-pass design)

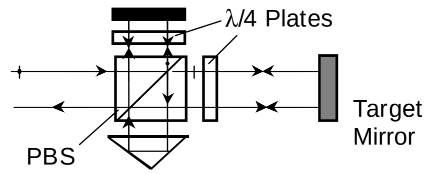


In the Plane Mirror Interferometer configuration shown above, the reference beam reflects at the hypotenuse of the PBS and is sent to a retroreflector. The beam reflects off the retroreflector, returns to the interferometer and exits the PBS parallel to the input beam.

The measurement beam transmits through the beamsplitter and exits the PBS through a quarter waveplate. The quarter waveplate changes the polarization state of the beam to circular. The beam travels to the target mirror and upon returning to the polarization beamsplitter passes back through the waveplate. The polarization state of the measurement beam is now at a polarization state that has been shifted by 90° with respect to its original state. Therefore, the beam reflects at the hypotenuse of the PBS, reflects off a retroreflector and makes a second pass to the target. The same magnitude of polarization changes happen with the second pass transforming the beam back to its original polarization state upon return to the PBS where the measurement and reference beams overlap.

The Plane Mirror Interferometer (PMI) has a resolution twice that of the linear interferometer because of the two passes the measurement beam makes to the target mirror. A linear displacement of the target mirror, z, results in an optical path change equal to 4z.

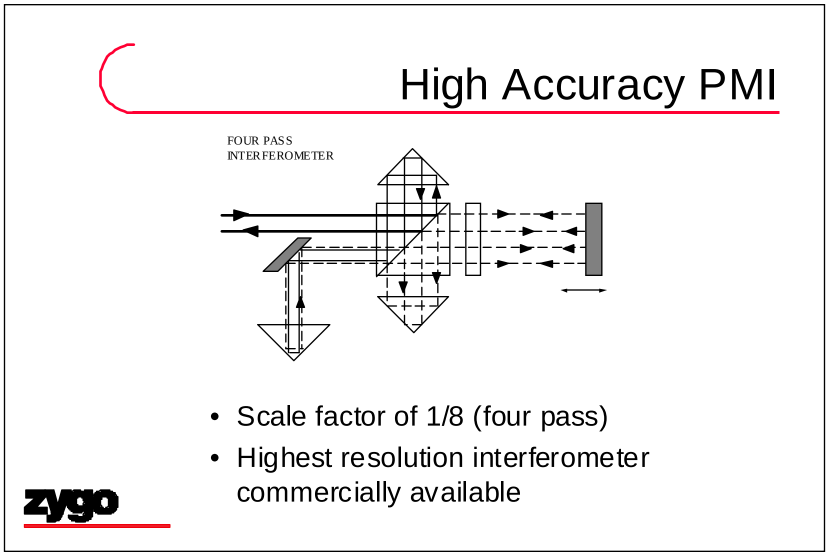
High Stability PMI



- Same resolution as PMI
- Thermally stable design; equal reference and measurement optical paths through glass

The High Stability Plane Mirror Interferometer (HSPMI) is less sensitive to environmental changes than a standard PMI due to its thermally stable design. The measurement and reference portions of the beam pass through equal amounts of glass. Therefore; changes in the environment will effect the portion of the beams in the interferometer equally.

Typical values for the temperature coefficient are 0.306 micrometers/degree C for a Plane Mirror Interferometer and 0.018 for a High Stability PMI design.

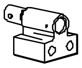


The four pass interferometer shown above is a variation of a Plane Mirror. The beam that would be the output in a two pass PMI is intercepted by a retroreflector that sends the beam back through the interferometer for two more passes to the target. The resultant resolution is two times better than that of a two pass interferometer.

The above example is not thermally stable. Four pass designs with equivalent measurement and reference beam paths are commercially available.

Receiver Components

- Early vintage DMI's use receivers that mix the beams and convert the optical signal.
- Fiber optic receivers (pick-ups & couplers) have become the industry standard for transfer of optical signals.



- Detection built into measurement board or offline receiver.
- Fiber optic receivers eliminate heat, reduce cost, improve electrical isolation & minimize size.

The original DMI systems used bulky electrical receivers to convert the optical signal produced by the overlapping measurement and reference beams. The receivers had a tendency to produce heat and cause electrically induced errors.

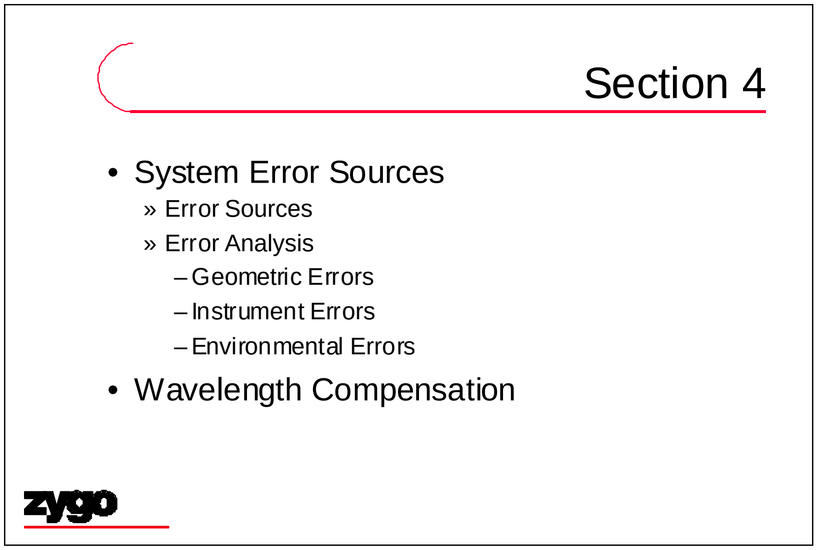
Fiber optic receivers were developed to eliminate the electrically induced errors and reduce cost and package size.

Fiber Optics

- Heterodyne
  - » Need to maintain two polarizations makes delivery difficult
  - » Reception is easy if analyzer is at fiber entry
- Homodyne
  - » One frequency, same polarization; delivery easy
  - » Need for quadrature detection makes reception difficult
  - » Must maintain polarization state independent of thermal and stress changes to fiber

Some AC systems utilize fiber optic technology for transmission of the measurement signal from the interferometer to the measurement board and for sending the reference signal frequency from the laser to the board.

Due to losses in the fiber delivery of a heterodyne signal is not efficient.



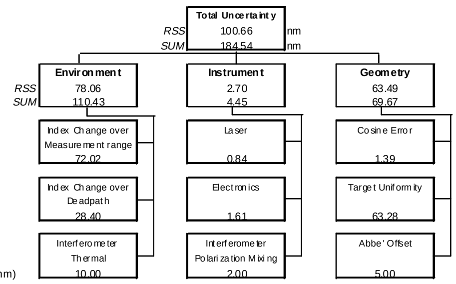
This section consists of a sample error analysis and discusses how error sources can be minimized or eliminated. The example error analysis is based on an application using a DMI with 1.24 nanometer linear resolution for the linear leg of a compact two-axis interferometer.

The metrology conditions for the example are shown below:

Temperature variation	1.0°C
Pressure variation	0.25 mm Hg
Humidity variation	10%
Range of motion	60 mm
Dead path distance	12.7 mm
Interferometer dead path	10.96 mm
Thermal coefficient	0.01 µm/°C
Laser stability	0.01 ppm
Electronics accuracy	1.3 counts (1.61 nm)
Polarization mixing	2 nm
Target mirror angle	5 µrad
Abbé Offset	1 mm
Target mirror flatness	$\lambda/10$ PV (63.2 nm)

Error Analysis

System Parameters		Units
Temperature Resolution	1	deg C
Pressure Resolution	0.25	mm Hg
Humidity Resolution	10	%
Measurement Range	60	mm
Dead Path Distance	12.7	mm
Interferometer Deadpath	10.96	mm
Inteferometer max temp coeff	0.01	um/deg C
Maximum laser instability	0.01	ppm
Electronics Accuracy	1.3	counts
Optical System Resolution	1.24	nm
Polarization Mixing	2	nm
Change in beam overlap	0.2	mm
Target mirror surface figure (p-v)	0.10	wavelength (1 wv = 633nm)
Abbe' Offset	1	mm
Target mirror Abbe' angle	5.0	urad (1urad=.21 arcsec)



Environmental errors are the largest contributor to most DMI systems. Controlling or monitoring the environment or minimizing the measurement time will reduce environmentally induced errors.

The target uniformity error can be minimized by mapping the mirror distortion and generating a software correction table.

Geometric Errors

Alignment/cosine error	1.39 (nm)	
Target uniformity	63.28	
Abbé errors	5.00	
Total (sum/rss)	**69.67**	**63.49**

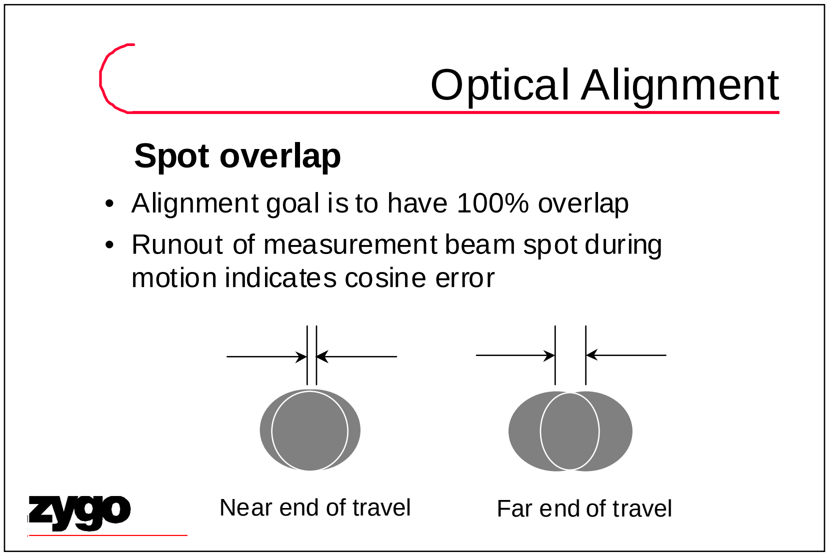
Geometric errors can be minimized by following a stringent setup and system alignment procedure and using a optically flat target mirror or compensating for a distorted target through a software look-up table.

Cosine error results from an angular misalignment between the measurement laser beam and the axis of motion.

The target uniformity error represents the error caused by non-uniformities in the target mirror.

Abbe error results from an offset between the measurement laser beam and the axis of motion of the part under test.

Typically, target mirror non-uniformity is the largest geometrical error source.

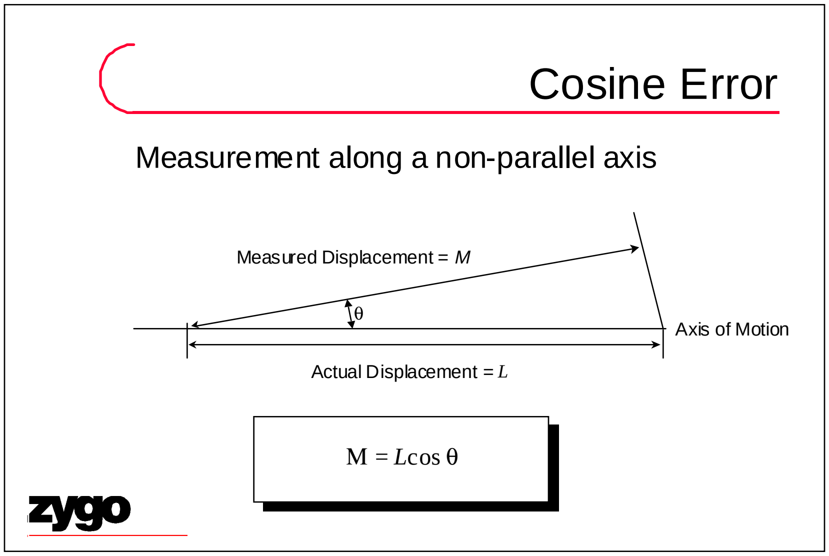


In a single axis system the spot overlap can be minimal and still yield a sufficient measurement signal (minimum overlap is approximately 50%). As the number of axes increases and the efficiency of the interferometers decrease, the overlap must be near 100%.

Alignment of the measurement beam parallel to the motion can be accomplished by observing the return spot of the measurement beam with respect to the reference beam. As the stage is moved, any angular error (cosine error) shows up as runout in the spot position.

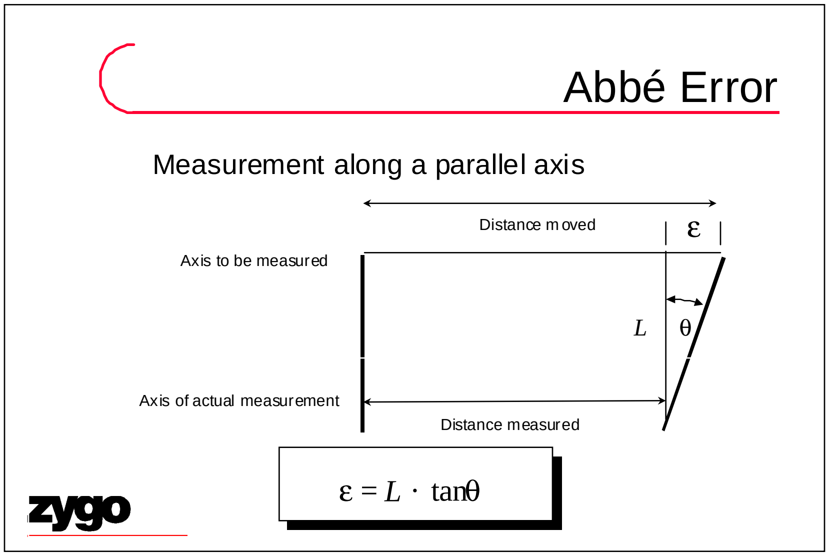
For example; Observing a 1 mm runout over a 1 m motion yields a 0.5 mrad alignment error.

$$a = \text{spot runout} / (2 \cdot \text{range of motion})$$



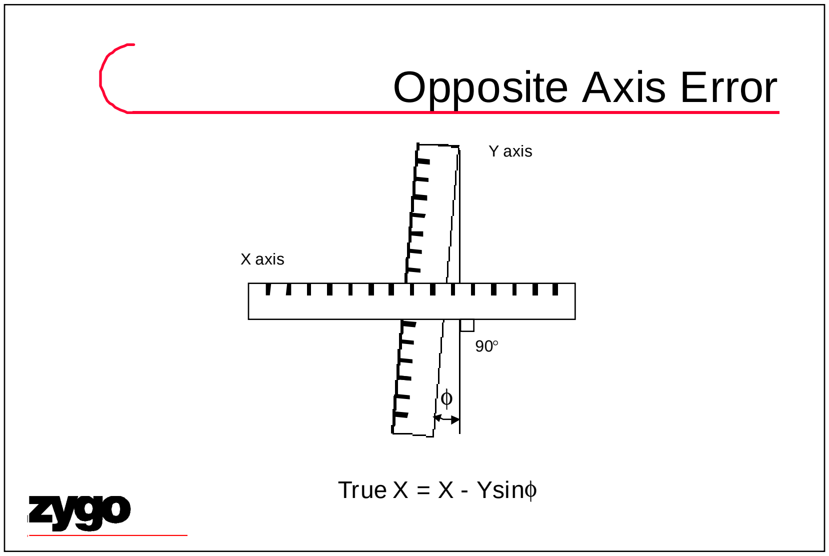
A cosine error results from an angular misalignment between the measurement laser beam and the axis of motion. The error is generally negligible until the angle becomes quite large.

Besides degrading the optical signal, a cosine error will cause the interferometer to measure a displacement shorter than the actual distance traveled.

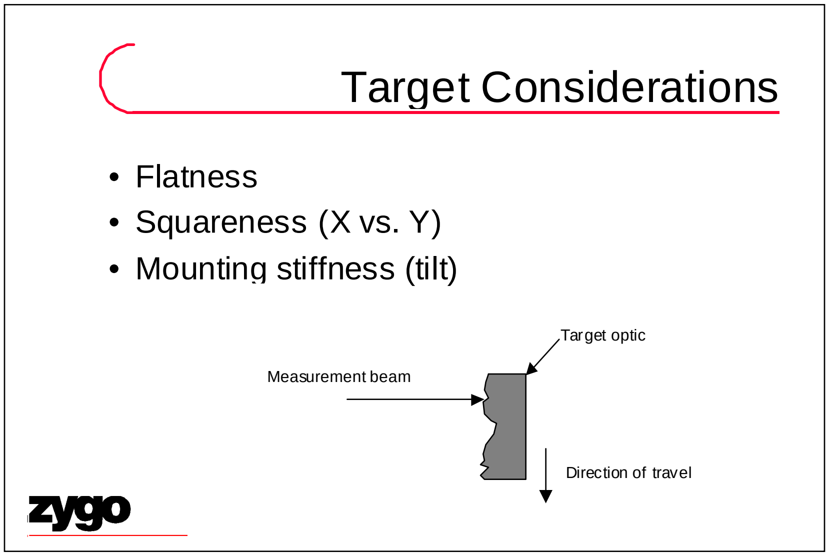


When the axis of measurement is offset from the axis of interest, Abbé errors will occur. As first described by Dr. Ernst Abbé of Zeiss:

“If errors of parallax are to be avoided, the measuring systems must be placed coaxially to the line in which displacement is to be measured on the workpiece.”



Opposite axis errors are often present in mechanical measuring systems. An opposite axis error is caused when perpendicular axes are not truly orthogonal to each other. This error is typically eliminated when a standard DMI system alignment procedure is followed.



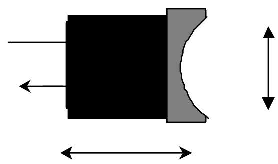
The target mirror must be flat to fractions of a wavelength in applications that require multiple axes of travel. A target mirror with a surface figure of $\lambda/10$ can contribute up to 63 nanometers of error as the stage travels along the axis parallel to the clear aperture of the mirror.

The target mirror normal must be aligned parallel to the axis of the stage travel to minimize tilt error. Tilt of the plane target mirror induces a change in the optical path difference. This error is proportional to the spacing between the interferometer and the target and is nonlinear with the tilt angle. The magnitude of this error can run from negligible up to a few hundred nanometers, depending upon the implementation.

Mechanical Stability

"Everything is Rubber"

- Vibration
- Target/part mount stiffness



Stiffness of the mechanical assembly is critical. If the physical relationship between the target optic and the point of interest changes during the measurement time, this is indistinguishable from actual motion.

Vibration effects can be minimized by taking several measurements at one position and averaging them together.

Instrumentation Errors

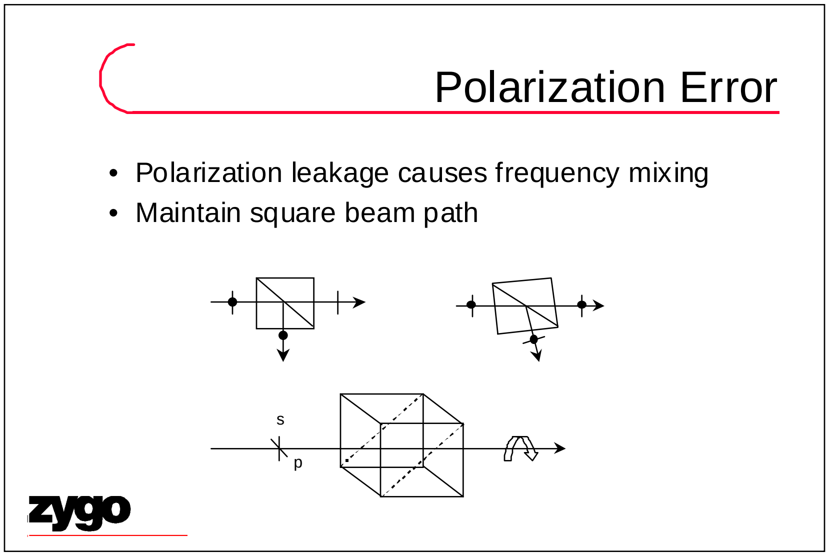
• Laser	0.84 (nm)	
» Laser stability		
• Electronics	1.61	
» Nonlinearity		
» Quantization (LSB)		
• Interferometer (optics)	2.00	
» Polarization mixing		
Total (sum/rss)	**4.45**	**2.70**

Instrumentation errors are not under the users control. These errors are based on the suppliers system parameters.

The basis of a DMI is the wavelength of the laser source. Stability circuitry within the laser head is designed to control the output frequency of the laser tube at a fixed value.

The contribution of the electronic uncertainty to the error analysis is a product of the electronic accuracy of the measurement board and the optical resolution of the interferometer.

Polarization mixing errors are caused by imperfections in the optical components and their coatings. This error can be minimized by optimizing the rotation of the interferometer about the optical axis. The magnitude of the polarization mixing error will increase if the optical alignment causes the incident beam not to lie perpendicular to the plane of incidence. Optical components with dielectric coatings are very polarization sensitive and can induce additional errors if not aligned properly.



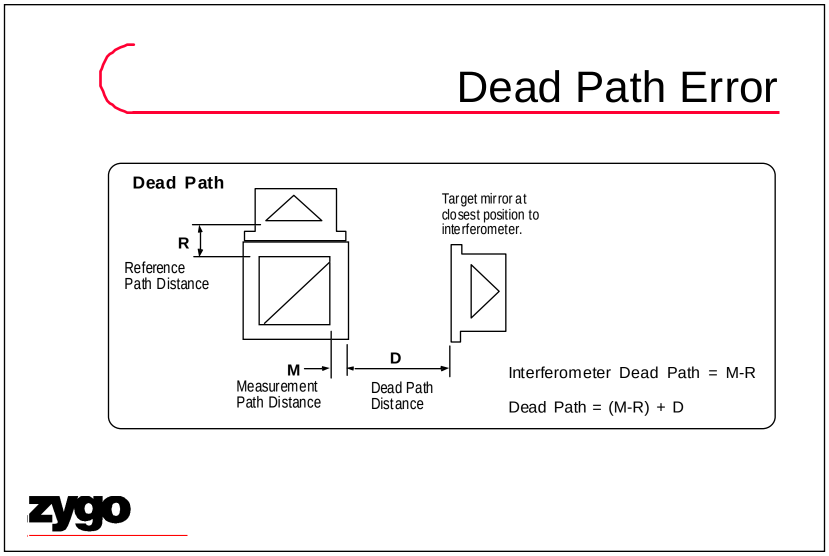
Polarization mixing of the laser’s frequency components within the interferometer causes a nonlinear relationship between the measured displacement and the actual displacement. To minimize errors caused by polarization mixing, a perpendicular relationship must be maintained between the two frequency components of the laser head and the orientation of the polarization-sensitive optical components.

The angular rotation of the interferometer about the optical axis should be limited to less than 1 degree to minimize polarization errors.

Environmental Errors

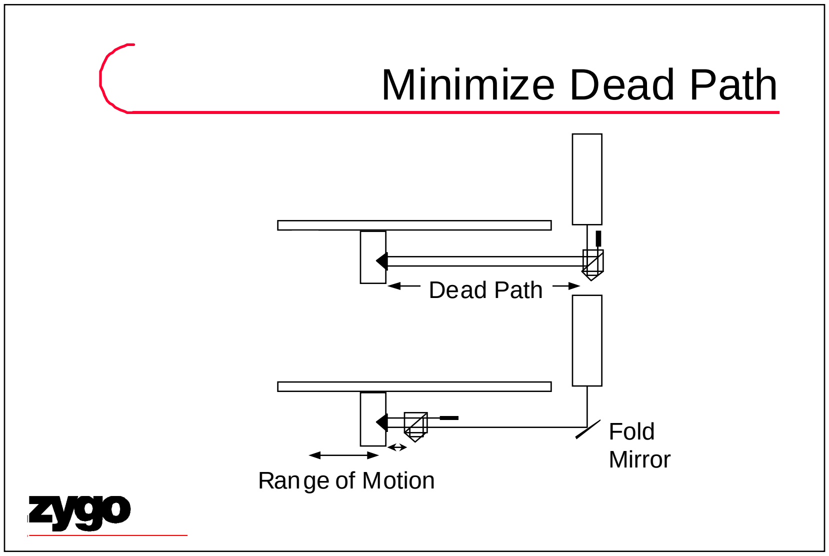
• Air index change » Measurement range	72.02 (nm)	
• Dead path error » Index change effects	28.40	
• Interferometer thermal » Thermal expansion	10.00	
Total (sum/rss)	**110.43**	**78.06**

Environmental errors are usually the largest contributor to a DMI error budget. Variations in the index of refraction of the air alter the wavelength of the laser source and change the apparent length of the optical path. The index of refraction changes with deviations in the temperature, pressure and humidity.

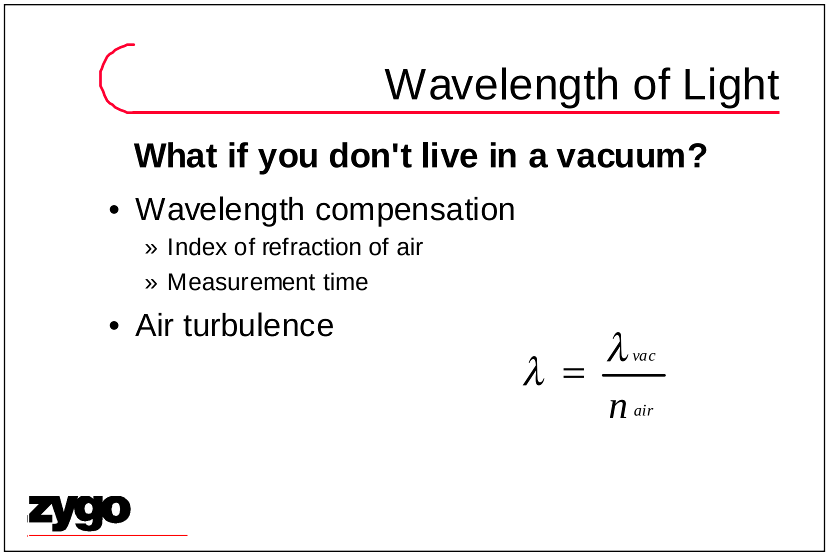


Dead path is the difference in distance in air between the reference and measurement paths of an interferometer configuration. The dead path error is caused by a change in the environment during the measurement. To minimize dead path distance, locate the interferometer as close to the target mirror as possible. Minimizing environmental changes during the time of the measurement also reduces the dead path error.

If the dead path distance is known, and there is active wavelength compensation, either by Edlèn’s equation or using a refractometer, then the dead path error can be corrected.

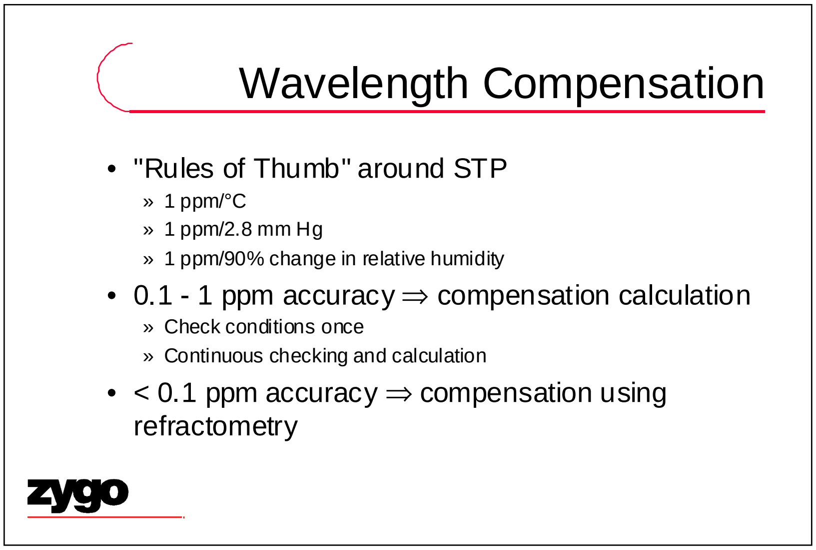


The upper example is using a right angle configured interferometer that is positioned a long distance from the target's travel. The lower example shows how adding a fold mirror and changing the interferometer to a straight through configuration can minimize the potential dead path error.



Unless measurements are taken in a vacuum, accurate displacement calculations can only be made if the measurement beam vacuum wavelength, λv, is divided by the index of refraction of air, λv/n. For nominal conditions (pressure = 760 mm Hg, temperature = 20°C, humidity = 50%), the index of refraction is 1.000271296. Depending on the requirements in the specific application, the index of refraction can be measured or can be set to some nominal value determined by the user.

Air turbulence is movement of thermal gradients in the air through the beam path. The magnitude of the air turbulence effects can be large if precautions are not taken. The simplest precaution is to place tubes along the beam path, except where there is actual motion. More extreme, and effective, methods include operating in a helium atmosphere or operating in a vacuum.



STP = Standard Temperature and Pressure

T = 20°C

P = 760 mm Hg

RH = 50%

Compensation Formulae

Edlèn's equation and variants

$$n = 1 + (3.8369 \cdot 10^{-7} \cdot P)\left[\frac{1 + P \cdot (0.817 - 0.0133 \cdot T) \cdot 10^{-6}}{1 + 0.003661 \cdot T}\right] - 5.607943 \cdot 10^{-8} \cdot f$$

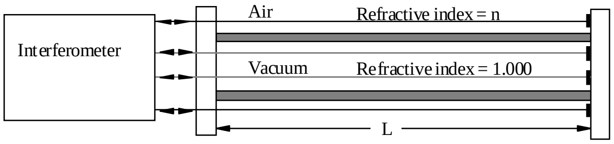
$$f = \frac{RH}{100} \cdot \left[4.07859739 + 0.44301857 \cdot T + 0.00232093 \cdot T^{2} + 0.00045785 \cdot T^{3}\right]$$

$$n = 3.836391 \cdot P\left[\frac{1 + P \cdot (0.817 - 0.0133 \cdot T) \cdot 10^{-6}}{1 + 0.003661 \cdot T}\right] - 3.033 \cdot 10^{-3} \cdot RH \cdot e^{0.057627T}$$

Changes in the environment over the time of measurement are typically the largest error source in a DMI metrology system. Controlling the climate, monitoring the pressure, temperature and humidity changes and/or reducing the measurement time will minimize these errors.

Edlèn published the first paper detailing wavelength compensation calculations. Shown above is Edlèn's formula with a power series expansion for the water vapor pressure term and an alternate formulation using an exponential fit. Other versions of Edlèn's formula exist. For more precise work, it is possible to incorporate molecular concentrations of the air, such as the partial pressure of CO_2, into the calculation.

Refractometry

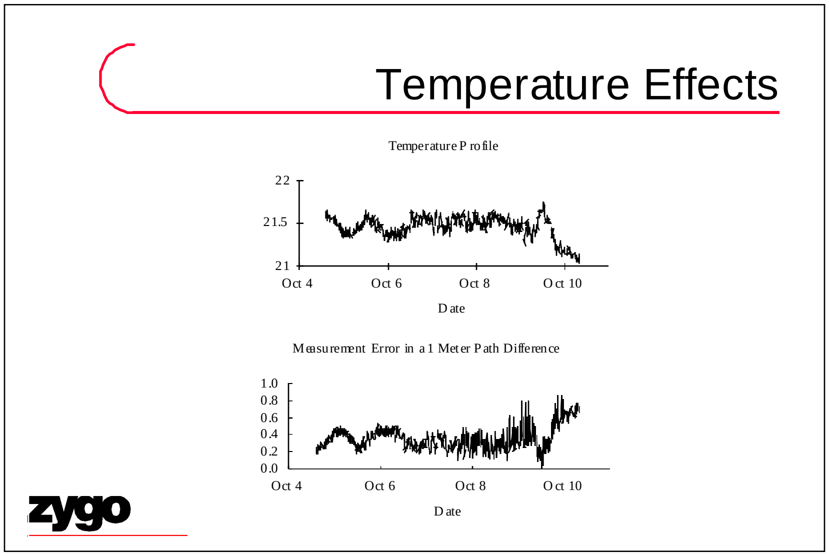


$$OPD_0 = 4L(n_0 - 1) \qquad OPD = 4L(n - 1)$$

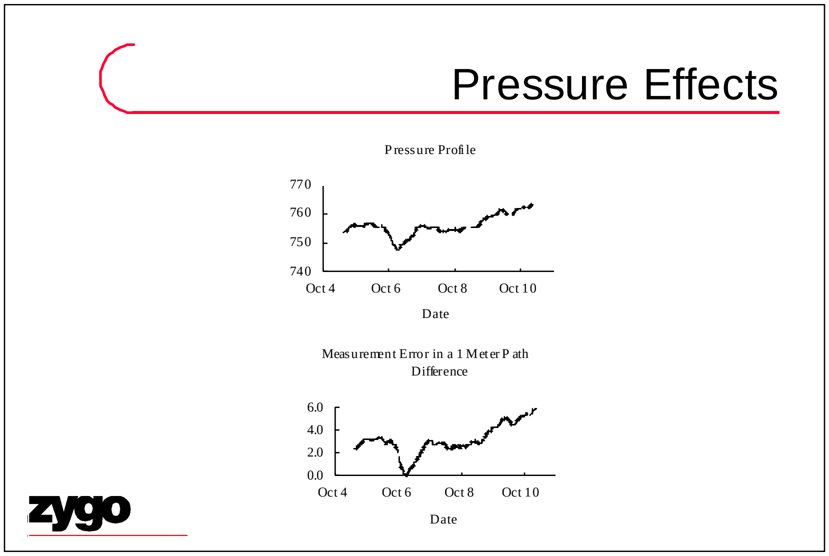
$$\Delta OPD = 4L(n - n_0)$$

An optical wavelength compensator (refractometer) measures the change in the refractive index of air. Since it measures relative change, it is important to know the index of refraction at the start of the measurement, no. This may be accomplished using Edlèn’s equations and taking initial measurements of the temperature, pressure and humidity.

The measurement and reference beams in the refractometer travel across the same nominal distance; the reference beam travels through a pair of vacuum sealed tubes while the measurement beam travels through air. The difference between the two represents the change in the index of refraction over the time of the measurement.



In this example the temperature was monitored for a period of six days. Edlèn's equation was used to calculate the error in a test setup with a one meter optical path difference between the measurement and reference beams. The pressure was assumed constant at 760 mm Hg and the relative humidity was taken as 50%.



In this example the pressure was monitored for a period of six days. Edlèn's equation was used to calculate the error in a test setup with a one meter optical path difference between the measurement and reference beams. The temperature was assumed constant at 20°C and the relative humidity was taken as 50%.

Section 5

- DMI Applications
- Static Applications
  - » Radius of Curvature
  - » Machine Tool Calibration
- Dynamic Applications
  - » Wafer Processing
  - » PZT Calibration

Static Applications acquire position data on a point by point basis. Examples of static measurements include the measurement of the radius of curvature of an optic and calibrations of machine tools and micrometer driven stages.

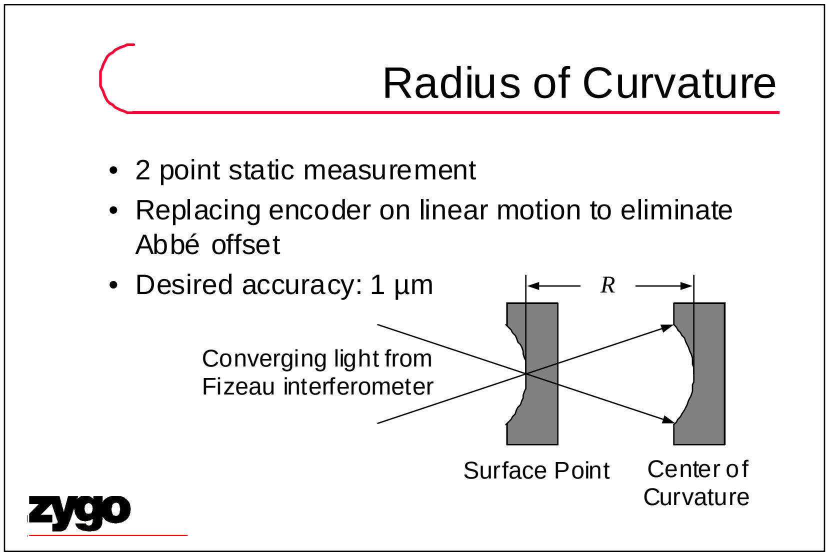
Dynamic applications involve the active monitoring a process. Vibration measurements, calibration of the motion of a piezoelectric transducer (PZT) and closed loop stage control are typical dynamic applications.

Typical DMI Applications

Calibration (static)
- Machine tools
- CTE
- Angular motion
- X-Y stages

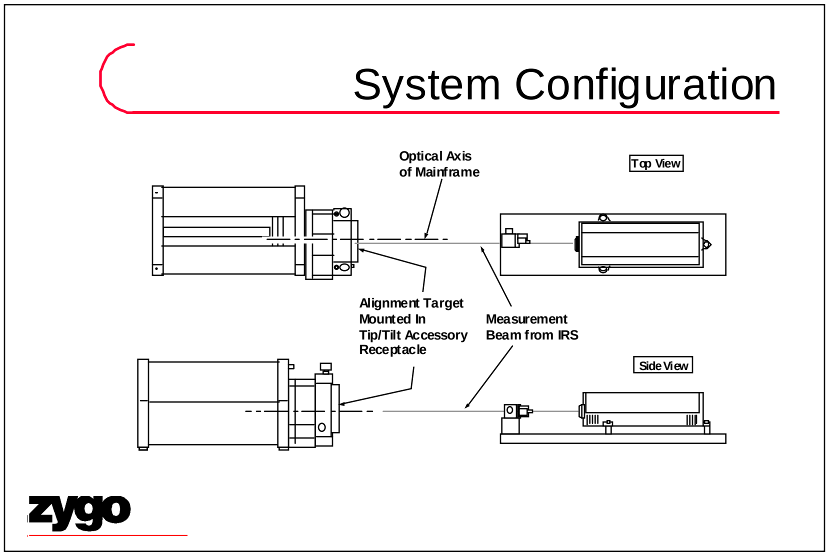
Production (dynamic)
- Lithography instruments
- Diamond turning
- Precision processing
- X-Y stage control

zygo



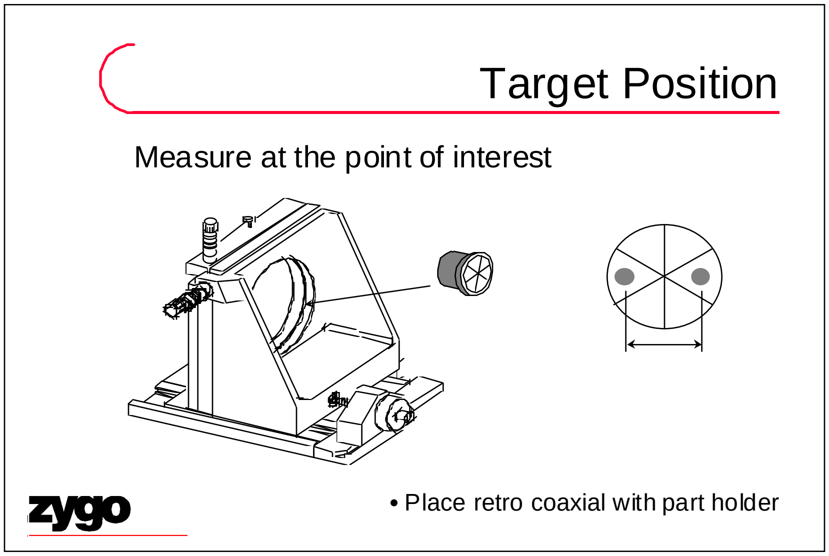
The accurate measurement of the radius of curvature of a spherical surface is accomplished using a DMI and a Fizeau interferometer. The Fizeau interferometer is used to determine the locations of the center of curvature and a point on the surface of the sphere. As the sphere is moved between the two positions, its motion is tracked by the DMI; the result is the radius of the optical component.

In some implementations the motion of the optical mount is motorized, but in most, the motion is provided by the metrologist pushing a mount which holds the spherical component along a guide bar.



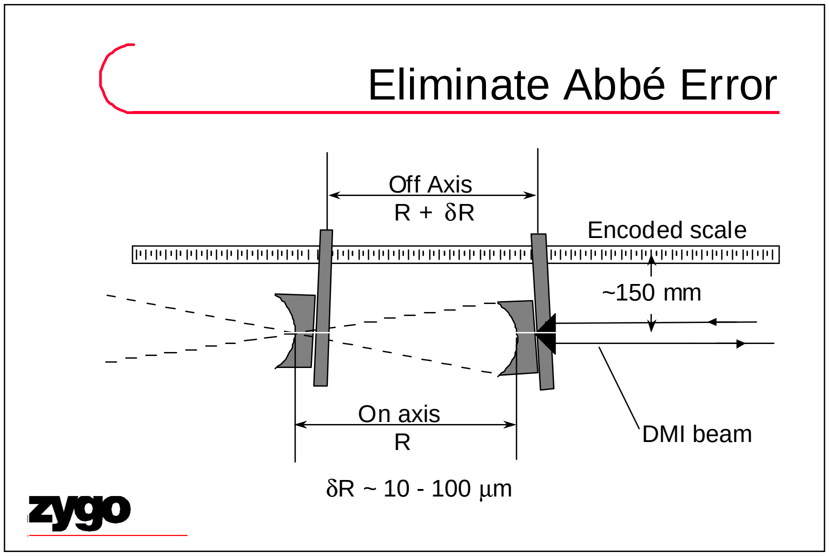
In the implementation, the motion of the spherical test piece can be driven by either a motorized stage, or by manual motion on the part of the metrologist. In the latter case, the mount which holds the part may tilt or move slightly off axis during the motion. Another concern about manual motion is that an operator's hand may break the DMI beam, causing a measurement error. This interferometer is not robust to lateral shift of the retroreflector during motion. Motion of half the beam diameter causes the beam to shift by the full diameter.

Either a Heterodyne or Homodyne system can be used for a radius measuring application. If a Homodyne system is used it will require direction sensing capability since the operator will most likely adjust the position of the optic back and forth to optimize the alignment.



Alignment of this system must insure the following:

1. Measurement at the point of interest (center of the optic).
2. The test and reference beams overlap enough to yield a strong enough signal.
3. The test beam runs parallel to the axis of motion.
4. The target tilts with the optic as fine alignment is performed.

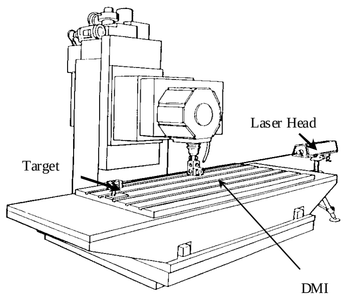


One of the main reasons for using a DMI in a radius measurement is the elimination of Abbé error. Abbé error results from an offset between the axis of the measurement laser beam (R) and the axis of motion of the part under test.

The magnitude of Abbé errors is a function of the mechanical integrity of the mount and rail assembly. For a large setup, as shown, there is enough mechanical slop so that the Abbé errors will typically be in the 10 - 100 µm range. A smaller, integrated assembly, as would be useful for radii <50 mm, can be built more stiffly and the Abbé errors can be brought down into the 1 - 10 µm range.

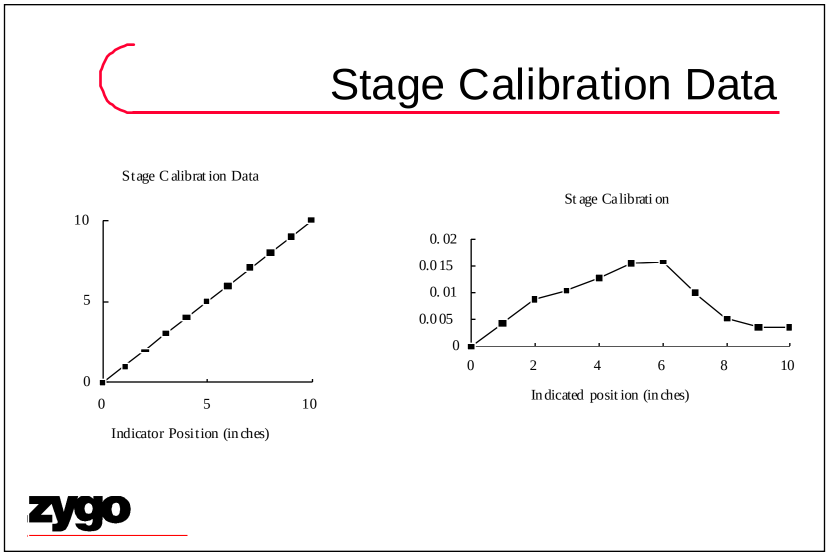
Machine Tool Calibration

- Interferometer mounts on machine head
- Often done with DC systems
- Motion linearity
- Cosine error correction



zygo

Machine tool calibration is a static application with many sample points. Here the position of the machine tool is measured and compared with the machine programmed position. A correction table can be formed or a correction function can be fit to the measured data.



The stage calibration data can be used to create a look-up table to correct for errors inherent in the machine. The first plot shows the position as measured by the DMI as a function of indicator position. The difference between the indicator position and DMI position define the error in the machine. The second plot is the resultant stage calibration data.

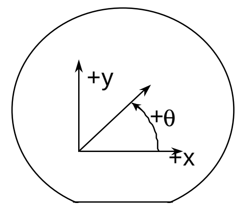
Dynamic Applications

- Noise less an issue because of fast samples
- Good coupling of time vs. position important
  - » Dynamic error
- Numerical derivatives for velocity and acceleration can be noisy
- Triggering to events may be required

A dynamic measurement measures the displacement path during the motion of the target as opposed to the static application that measures the target after it has stopped.

Wafer Processing

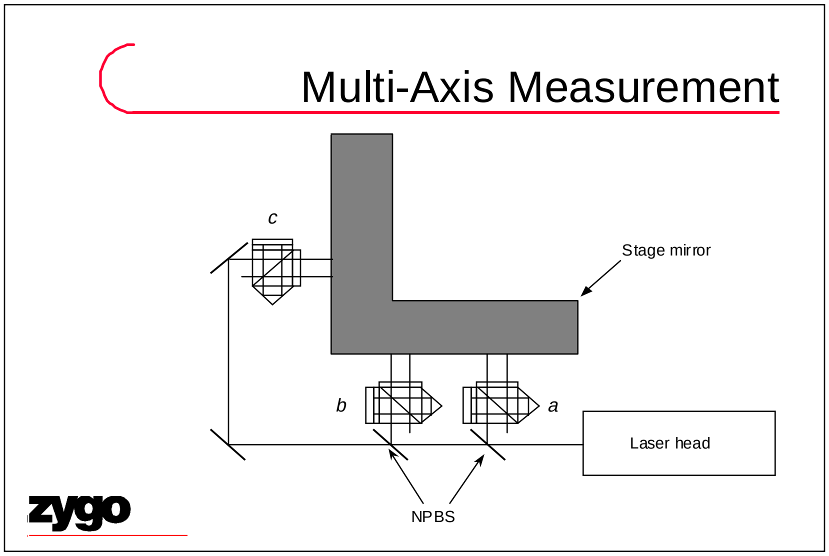
- Accurate monitoring of X-Y position and in-plane rotation
  - » Calibration
  - » Control
- Multi-axis measurement
  - » Error detection
  - » Energy budget
- Orthogonal motion



The machine control application is a case of the DMI designed in as a metrology subsystem for a larger machine. The maximum number of axes which can be monitored is determined by the system power budget and electronics limitations. Some DMI systems have a limit to the number of axes the electronics can control. Each axis of measurement requires a minimum amount of energy at the detection electronics. A power budget based on the minimum optical efficiency levels of the individual components is an integral part of the DMI system design process.

In any DMI system, it is essential that the control device knows if the interferometer stops measuring in one of the axes, even if only momentarily. If the signal is lost, or if the velocity exceeds the electronics bandwidth, there is an error. The DMI must be able to detect errors. Error detection is trivial in a heterodyne system. If the reference or measurement signal has two rising edges before the other has one, then an error occurs. Thus, the system is checking for errors at the heterodyne shift frequency. In a homodyne system, this error is less obvious.

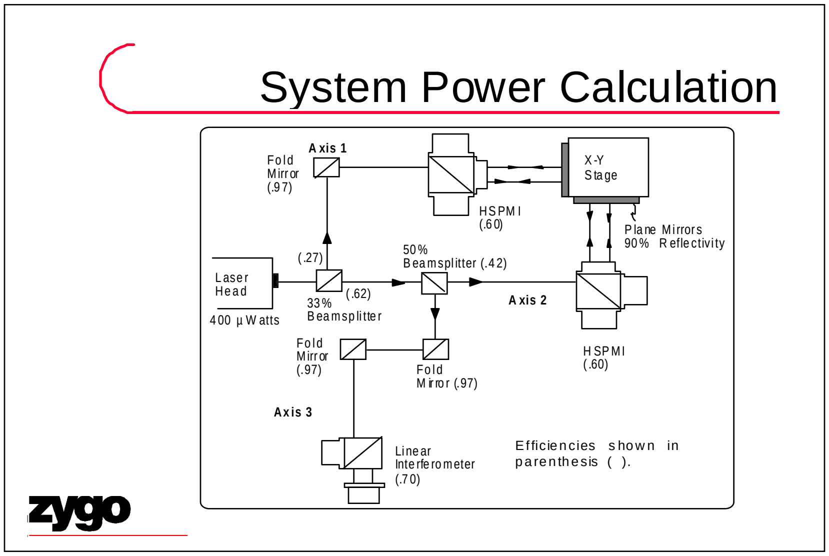
The energy required at the detector is significantly less for the heterodyne systems. It requires only two or three bits of signal to noise ratio to detect the heterodyne signal. In the homodyne case, many more bits of intensity detection are required on at least two if not as many as four detectors. This results in a significantly larger energy requirement per axis.



In a multi-axis system, care must be taken to optimize the power distribution. The non-polarizing beamsplitters (NPBS) should be designed to equalize the light intensity in the three measurement axes.

In the above example the NPBS feeding interferometer “A” should reflect 33% and transmit 67%. The next beamsplitter in the path should be a 50/50 splitter and the following two elements are fold mirrors.

NOTE: The above beamsplitter choices assume that interferometers “A”, “B” and “C” have the same optical efficiency. Different types of interferometers will have different efficiencies.

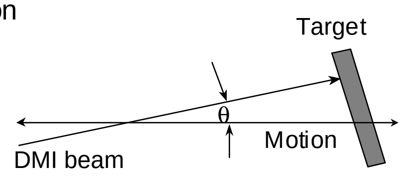


The detection electronics portion of a DMI system requires a fixed amount of optical power to achieve a detectable signal. Using optical components with optimum efficiency and target mirrors with sufficient reflectivity play an important role in designing a Displacement Measuring Interferometer system.

Some multiple axis configurations require more optical power than a single laser source can provide. Due to phase differences in the reference frequency from one laser to the next, synchronizable laser heads are required for applications that require more power than a single laser can produce.

Design Considerations

- Turbulent airflow (clean room)
- Wavelength compensation
- Beam profile quality
- Cosine error
- Bulk thermal changes
- Slew rates (control)
- Vibration (control)



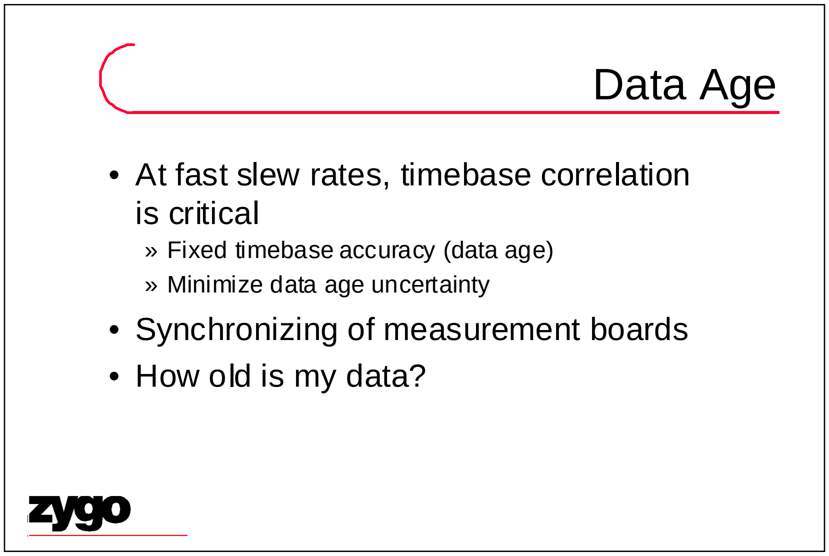
Turbulent airflow can be reduced by sending the DMI laser beam through tubes.

Wavelength compensation may not be required if the environmental variations over the time of the measurement are minimal.

Poor quality optics can degrade the beam quality and directly effect the ability to produce an accurate measurement.

Cosine error is introduced by misalignment of the laser to the axis of motion. This error can be detected by looking for lateral translation of the beam spot on the target mirror during motion.

The maximum slew rate which the DMI can track will vary with the scale factor of the optical configuration. A configuration with a scale factor of 1/2 can track twice the velocity of a configuration with a scale factor of 1/4.

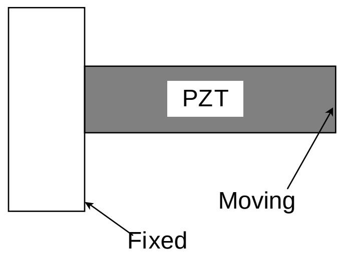


To accurately control precision motion it is necessary to provide not both position and time data. Data age is defined as the difference in the time between when the object of interest is measured and when the user control system gets the position information. Data age uncertainty is defined as the maximum variation in the data age in a multi-axis system, due primarily to process variation in the electronics.

Having minimum data age and data age uncertainty is critical for multi-axis high velocity applications.

PZT Calibration

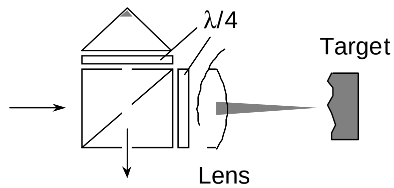
- Dynamic measurement
- Hysteresis
- Motion non-linearities
- Velocity calculation
- Vibration



Optical Probe

Displacement Measurement

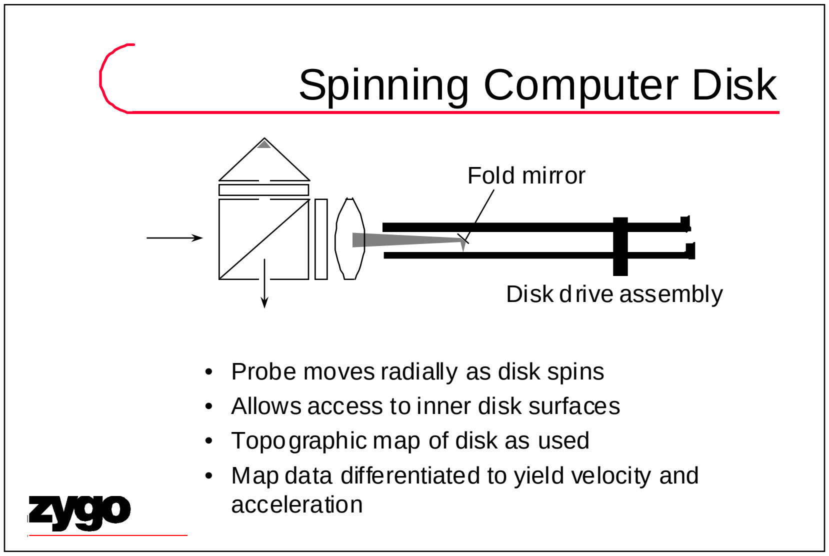
- Target tilt produces (nearly) constant beam offset
- Target tilt limited by lens NA
- Depth of focus limited by lens NA
- Target can be translated perpendicular to beam
- Target size < 1 mm



The optical probe allows measurement at a small spot: 65-100 µm diameter. The range of motion is limited by the depth of focus of the objective lens. Thus, the lens should be optimized for the application.

This interferometer is unusual, because it is aligned with no axial offset for the retro. The reference beam is centered on the apex of the retro and the measurement beam is centered on the objective lens.

The optical probe can accommodate a wide range of target reflectivity.



This application is used to actively measure the changes in the disk surface height as it is rotating within its drive assembly. Due to reduced spacing between the disks, this application is typically limited to measurements of only the top disk in the stack.

Other Dynamic Applications

- Monitoring chicken embryo heart rate...
  ...in the shell
- Monitoring the bending of nuclear fuel rods while welding tubes
- Measuring the growth of a bubble

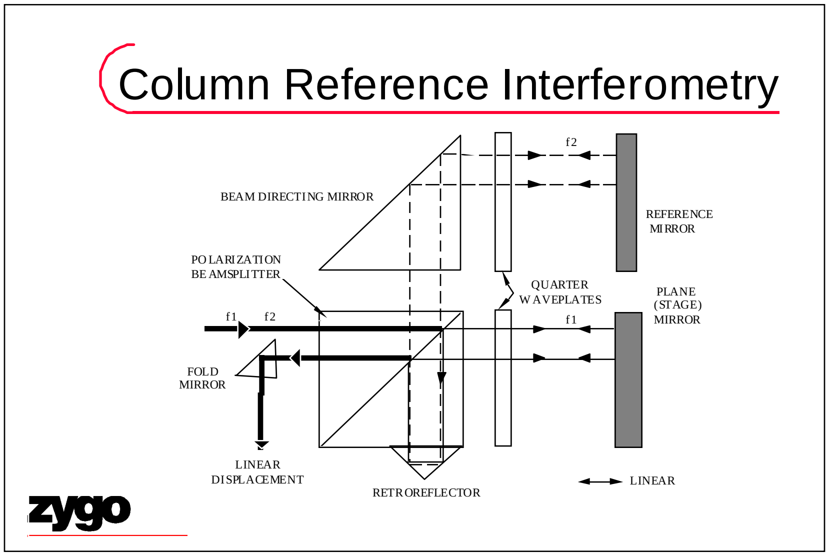
DMI Applications Summary

- Setup is critical to good results
- Minimize geometry errors by design
  - » Abbé
  - » Cosine
  - » Dead path
- Measure when environment is stable
- Minimize total time of measurements
- Compensate for air index of refraction
- Measure multiple degrees of freedom simultaneously

zygo

Section 6

- Application Specific Interferometers
  - » Column Reference
  - » Multi-axis Designs
  - » Angle Measurements
  - » Differential Interferometers
  - » Straightness Interferometer
  - » Summary

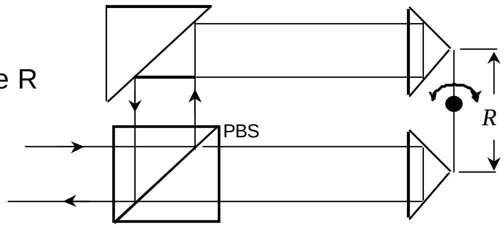


A column reference design allows for the measurement of the relative displacement between two active mirrors, while reducing a system's dead path error. The configuration shown depicts a two-pass column reference design. The resultant measurement is a linear displacement that represents the difference between a reference and a target mirror.

Angular Motion – Linear DMI

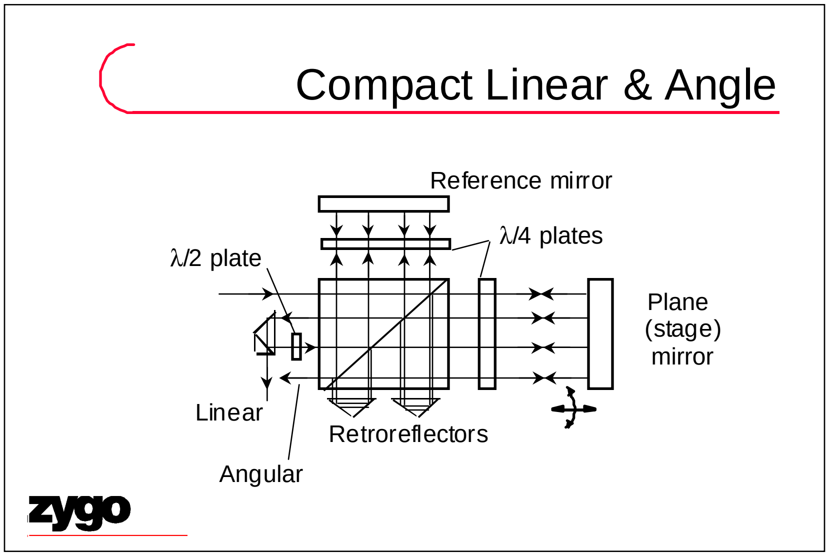
- DOPD evaluated over a baseline distance
- Angular resolution "unlimited"
- Not equal path
- Must determine R accurately

$$OPD = R \cdot (\sin\theta_1 - \sin\theta_2)$$



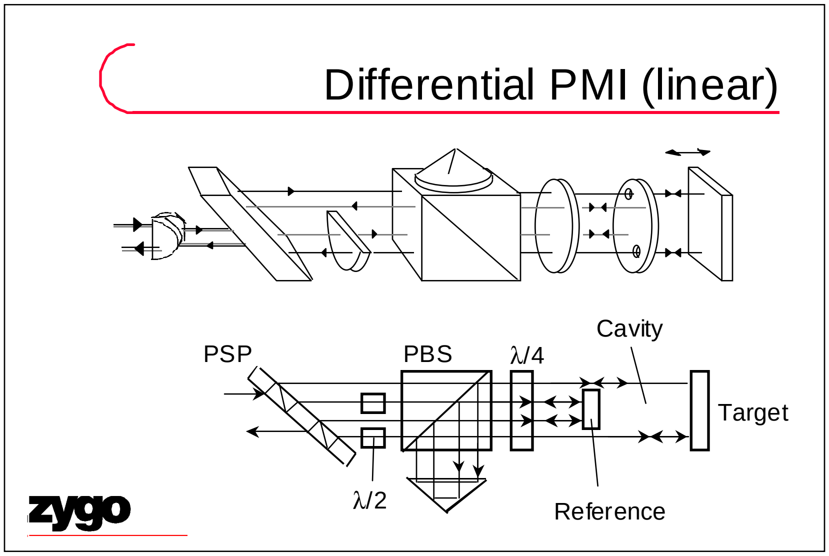
In this configuration, the reference beam is sent to one retroreflector and the test beam is sent to the other. The angles indicated in the equation above are the angles between the beams and the axes of the retroreflectors.

To achieve optimum accuracy, it is important to know the starting angle and the baseline distance, R, accurately.



The compact high stability interferometer uses 1/2 inch optics and a 3mm diameter source beam. Compact interferometer designs are typically monolithic optics assemblies that have a smaller footprint than the conventional interferometers that use 1 inch optics.

The two axis compact shown above outputs linear and angular displacement information through Optical Differencing. This technique uses the measurement beam from the linear axis as the reference beam for the angular axis. The result is a direct linear and angular displacement measurement of the target mirror.



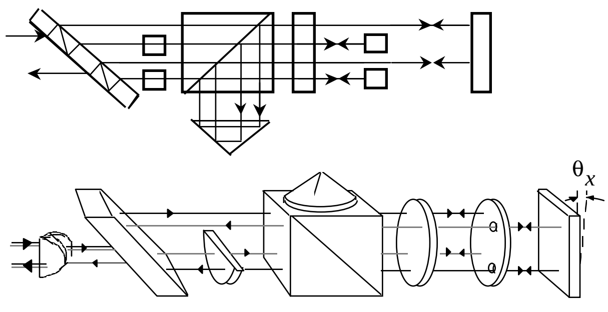
The Differential Plane Mirror Interferometer (DPMI) can be used to measure linear displacements or small angular displacements.

The light entering in the interferometer is split into two parallel beams by the polarization shear plate. The polarization of one of these beams is then rotated by a half wave plate to match the polarization of the other beam. In this way, the two beams travel the same path through the interferometer except that they are now displaced by a few millimeters.

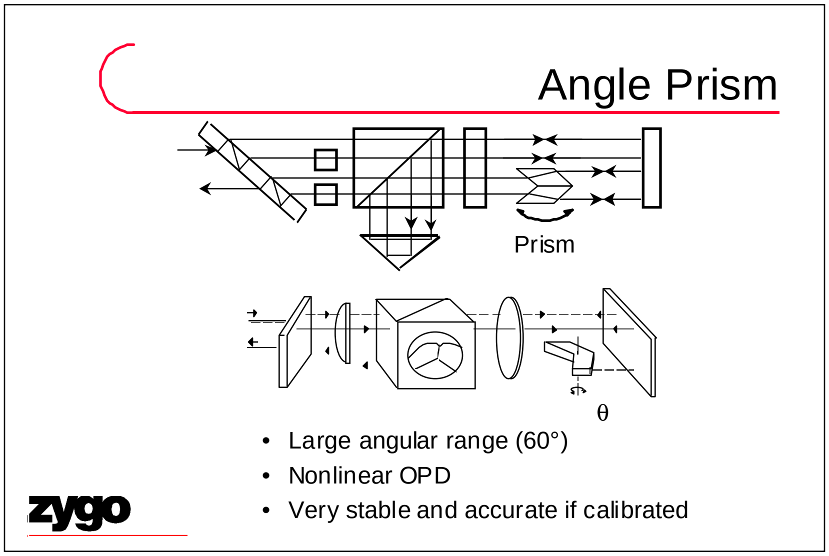
The reference mirror is fabricated with holes in it so that the measurement beam passes through the mirror and the reference beam is reflected. Each beam travels to/through the reference mirror twice and the beam footprint is a square arrangement of non-overlapping spots. In the linear displacement application the two paths of the measurement beam are oriented diagonally across the square pattern.

This interferometer design is good for remote or vacuum applications and has minimal thermal effects. However, it tends to be expensive due to the number and type of components required.

Differential PMI (angular)



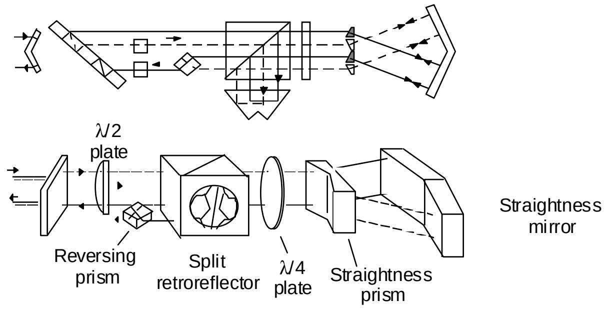
The DPMI can be configured for an angular measurement by simply changing the reference mirror. The angular displacement reference mirror has two holes that are positioned such that the second pass of the beams is directed to the opposite mirror it reflected from on the first pass. The resultant output is and angular displacement whose resolution is based on the separation between the beams on the target mirror (less than 0.1 arc second).



The angle prism must be used with the DPMI to measure an angular range of up to 60 degrees. Unlike other angular displacement configurations, the angle prism requires that a fixed mirror be placed between the interferometer and the prism.

The angle prism beam path consists of a pair of beams that travel to the mirror through air and a second pair that travel to the same mirror but through the calibrated angle prism. The prism calibration data is then used to calculate the angular displacement.

Straightness Interferometer



- Measures motion orthogonal to primary motion
- Resolution = sinθ of straightness mirror

zygo

As the straightness mirror moves, its surfaces remain normal to the beams. As the mirror is translated across the direction of motion, there is a change in optical path difference detected by the system.

The straightness prism and straightness mirror are designed for the particular application. A long travel requirement would either require a small angle deviation by the prism, or would require a vary long mirror. A straightness mirror with 2° tilts of the facets is typical.

Summary

Displacement Measuring Interferometers

- High Accuracy Displacement Measurement
- High Resolution
- High Velocity
- Eliminates Geometric Errors
  - » Abbé & opposite axis error
- Measures along the axis of motion
- Configurable for many measurement geometries
- Measures multiple axes simultaneously

GLOSSARY

Accuracy – The maximum deviation of a measurement from a known standard.

Abbé Principle – Abbé errors are introduced when the measurement axis is not coaxial with the axis to be measured. The Abbé Principle states that the measuring system must be placed coaxial to the line in which the displacement is to be measured.

Coherence – The phase relationship between any two points in a light beam is time-independent. The laser is a coherent light source because it produces radiation with waves vibrating in phase.

Collimation – The process by which a divergent beam of radiation or particles is converted into a parallel beam. All rays from a given field point are parallel. Highly directional beam propagation.

Dead path – The difference in distance in air between the reference and measurement optical elements and the polarization beamsplitter when the interferometric measurement is initiated.

Displacement – The difference between the initial position of an object and a later position of the same object.

Doppler Effect – A change in frequency of a wave (light) due to movement of the source of receiver. When the source and/or the receiver of the waves are in motion with respect to each other, the frequency increases or decreases according to the speed at which the distance between them is decreasing or increasing.

Heterodyne Detection – A Heterodyne Interferometer is also referred to as a two frequency or AC system. A heterodyne system uses a dual frequency source and detects changes in displacement by detecting a change in phase between the reference and measurement beams.

Homodyne Detection – A Homodyne Interferometer is also referred to as a single frequency or DC system. A homodyne system measures displacement of a target by detecting a change in beam intensity.

Interferometer – An optical arrangement where a beam of light from a light source is split into two beams by a beamsplitter, and the two beams are subsequently recombined after traversing different optical paths. The two recombined beams can then produce optical interference.

Laser – An acronym for “Light Amplification by Stimulated Emission of Radiation”. A laser provides monochromatic light of much greater coherence, collimation, and intensity than a thermal source.

Monochromatic Light – Light having only one wavelength.

Non-Polarizing Beamsplitter (NPBS) – Splits the source beam into two portions (reflected and transmitted). The reflected and transmitted components of a NPBS contain the same polarization characteristics of the source beam.

"P" Polarization – Describes the direction of polarization as referenced to an optical element when the incident light or beam is polarized parallel to the plane of incidence.

Path-length Compensated – The optical path lengths of the measurement and reference beams in glass are equal. Therefore, no change in optical path difference occurs between the measurement and reference beams if the interferometer temperature changes uniformily.

Plane of Incidence – Describes the plane at which light strikes a particular surface. It is the plane containing the direction of propogation of the incident light and the normal to the surface at the point of incidence.

Polarizing Beamsplitter (PBS) – Separates the source beam into two equal intensity beams that have unique polarization characteristics. The two beams generated by the PBS become the reference and measurement beams of an interferometer.

Polarization Coding – To split the input beam and to recombine the two beams in an interferometer using the polarization properties of the beams and optics.

Resolution – The smallest measurable increment. The units of resolution are units of length including fractions of lambda (λ). "λ" is the symbol for the wavelength of light. For a Helium-Neon laser lambda is 632.9 nanometers.

Retroreflector – An optical element that reflects a light beam parallel to the incident light beam regardless of the angular alignment with respect to the incident beam.

"S" Polarization – Describes the direction of polarization as referenced to an optical element when the incident light or beam is polarized perpendicular to the plane of incidence. "S" denotes perpendicular, fro the German "senkrecht".

Single Pass Interferometer – An interferometer in which the measurement beam reflects once from the moving optical element (retroreflector).

Slew rate – The velocity of the moving optical element while displacement measurements are in process.

Two-Pass Interferometer – An interferometer in which the measurement beam is reflected twice from the moving optical element.

Zeeman Effect – A method of generating two frequencies from a laser. The Zeeman technique applies a magnetic field to a laser to create two frequency states.

REFERENCES

1. Bennett, S.J., "A double-passed Michelson interferometer", Optics Communications. Vol. 4 (1972) No. 6, pp. 428-430.

2. Bobroff, N., "Residual errors in laser interferometery from air turbulence and nonlinearity", Applied Optics 26, pp. 2676-2682 (1987).

3. Bobroff, N., "Recent advances in displacement measuring interferometry", Meas. Sci. Tech. 4(9), pp. 907-926 (1993).

4. Bryan, J. B., and D. L. Carter, "How straight is straight?", American Machinist, p. 61-65, December (1989).

5. Demarest, F., "High-resolution, high-speed, low data age uncertainty, heterodyne displacement measuring interferometer electronics", Meas. Sci. Tech. 9, pp. 1024 - 1030 (1998).

6. Dukes, J. and Gordon, G., "A two hundred foot yardstick with graduations every microinch", HP Journal Vol 21 #12, Aug. 1970.

7. Edlén, B., "The refractive index of air", Metrologia 2, p. 71 (1966).

8. Estler, W. T., "High-accuracy displacement interferometery in air", Applied Optics 24, p. 808 (1985).

9. Hariharan, P., Optical Interferometry, Academic Press, 1985.

10. Jones, F. E., "The refractivity of air", Journal of Research of the National Bureau of Standards 86, p. 27 (1981).

11. Schellekens, P., et. al, "Measurements of the refractive index of air using interference refractometers", Metrologia 22, p.279 (1986),

12. Slocum, A. "Precision Machine Design", Prentice-Hall, Inc. 1992.

13. Sommargren, G.E., "A new laser measurement system for precision metrology", Precision Engineering. Vol 9, No. 4, p. 179 (1987).

14. Steinmetz, C.R., "Laser interferometry operates at submicrometer level", Laser Focus World, pp. 93-98, July (1990).

15. Zanoni, C., "Differential Interferometer Arrangements for Distance and Angle Measurements: Principles, Advantages and Applications", VDI BERICHTE NR. 749, pp. 93 – 106, (1989).