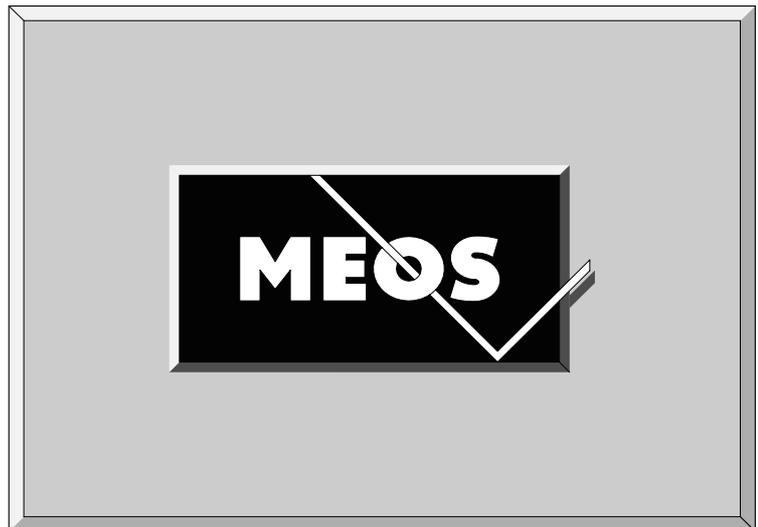
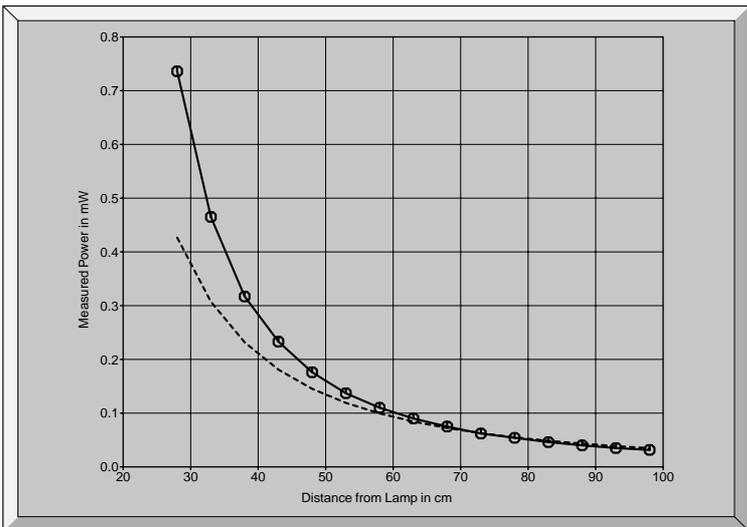
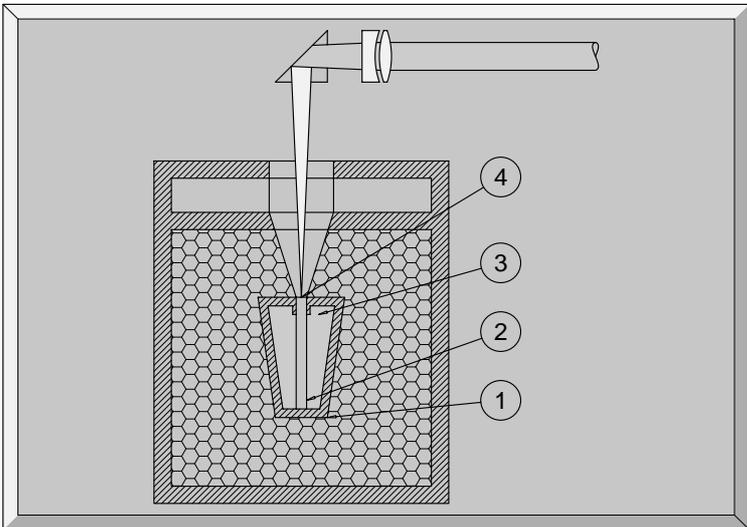
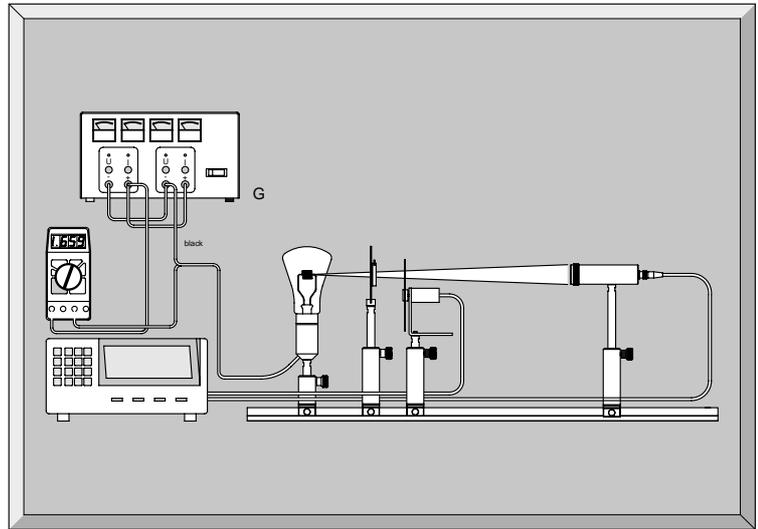
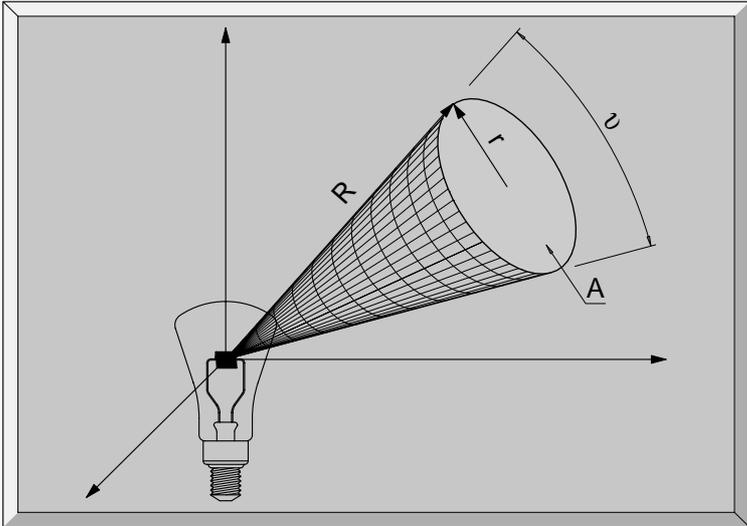


# Radiometry & Photometry



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# 1 Fundamentals

## 1.1 Introduction

Light, the giver of life, has always held a great fascination for human beings. It is therefore natural that people have been trying to find out what light actually is, for a very long time. We can see it, feel its warmth on our skin but we cannot touch it.

The ancient Greek philosophers thought light was an extremely fine kind of dust, originating in a source and covering the bodies it reached. They were convinced that light was made up of particles.

As human knowledge progressed and we began to understand waves and radiation, it was proved that light did not, in fact, consist of particles but that it is an electromagnetic radiation with the same characteristics as radio waves. The only difference is in the wavelength.

We now know, that the various characteristics of light are revealed to the observer depending on how he sets up his experiment. If the experimentalist sets up a demonstration apparatus for particles, he will be able to determine the characteristics of light particles. If the apparatus is the one used to show the characteristics of wavelengths, he will see light as a wave.

The question we would like to be answered is: What is light in actual fact? The duality of light can only be understood using modern quantum mechanics. Heisenberg showed, with his famous "Uncertainty relation", that strictly speaking, it is not possible to determine the position  $x$  and the impulse  $p$  of a particle of any given occurrence at the same time.

$$\Delta x \cdot \Delta p_x \geq \frac{1}{2} \hbar \tag{1.1.1}$$

If, for example, the experimentalist chooses a set up to examine particle characteristics, he will have chosen a very small uncertainty of the impulse  $\Delta p_x$ . The uncertainty  $\Delta x$  will therefore have to be very large and no information will be given on the position of the occurrence.

Uncertainties are not given by the measuring apparatus, but are of a basic nature.

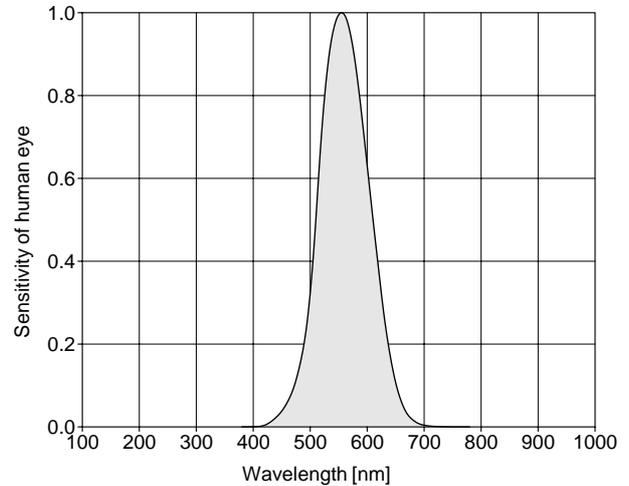
This means that light always has the particular quality the experimentalist wants to measure. We can find out about any characteristic of light as soon as we think of it. Fortunately the results are the same, whether we work with particles or wavelengths, thanks to Einstein and his famous formula

$$E = m \cdot c^2 = \hbar \cdot \omega \tag{1.1.2}$$

This equation states that the product of the mass  $m$  of a particle with the square of its speed  $c$  corresponds to its energy  $E$ . It also corresponds to the product of Planck's constant  $\hbar = h \cdot 2\pi$  and its radian frequency  $\omega = 2\pi \cdot \nu$ . In this case  $\nu$  represents the frequency of luminous radiation.

## 1.2 Types of luminous radiation

In the following, radiation always means electromagnetic radiation. We are particularly interested in the „optical range“. This range starts from wavelengths of 0.1  $\mu\text{m}$  to approx. 1 mm. The spectral range visible to the eye plays an important part.



**Fig. 1: Spectral sensitivity of the human eye**

Whereas the quantity of radiation in the optical range is usually measured in watt seconds or watts, the visible part of the optical radiation has its own photometric quantities with the basic unit known as candela (cd). It is the sixth variable apart from the basic quantities length (m), mass (kg), time (s), electrical current (A) and temperature (T), which were laid down at the 10th International Conference for Weights and Measures in 1954. In the following we will concentrate on the optical spectral range of electromagnetic radiation and first of all describe the basic regularities required for its deduction. In the next chapter, 1.3, we will examine radiation, for which the energy comes from the caloric content of a radiating body. This type of radiation is called thermal radiation and is emitted by glowing solids (metals, carbons, etc.) or gases at high pressures. But radiation can also be produced without the energy escaping from the caloric content of a medium, e.g. by introducing an electric energy (gas discharge).

In this type of radiation production, the temperature of the radiator (except in cases of loss mechanisms) does not change. Light produced in this way is known as cold light or luminescent radiation. The type of emission depends on the electronic or vibronic conditions of the atoms or molecules concerned. Light sources which emit light through these light mechanisms are an important part of modern light engineering. Whereas light bulbs are thermal radiators, fluorescent lamps, gas discharge lamps, screens and lasers are sources of cold light. We will discuss the regularities of this type of radiation in Chapter 1.4. In whichever way light radiation is produced, the aim is to determine the energy  $E$  of the radiation. But in light engineering, the characteristics of the light source itself must be quantified and made measurable, which will then make it possible for us to compare light sources at any time and in any place in the world. Just as there are comparative

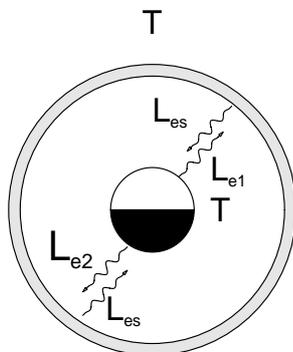
standards for mass, length, etc., there are also standards for light sources. The definition of the standards and its corresponding units and measurement procedure will be explained in Chapter 1.5.

### 1.3 Thermal Light

We know from our daily observations that a warm body emits light radiation. The higher the temperature of the body, the whiter the light. At the turn of the century, physicists were particularly preoccupied with this phenomenon. They were looking for the function giving the light energy of a radiant body as a function of the light wavelength  $\lambda$  and the temperature  $T$ . They presumed that the radiation energy was also influenced by the material characteristics of the body in question. Let us call these qualities  $P_{body}$ . The function

$$E(\lambda, T, P_{body}) = ? \tag{1.3.1}$$

had to be found which corresponded to the experimental data. If we consider the numerous substances which would radiate light when heated, it would at first seem impossible to derive a universal expression for  $E(\lambda, T, P_{body})$ . In 1861 G. Kirchoff made a crucial contribution by simplifying the rational examination of radiation. The experimental set up shown in Fig.2 helps us to understand this extremely important Kirchoff's law. The diagram shows a radiopaque covering inside which there is a ball, with a polished upper half and a blackened lower half. The inside of the covering is empty. This is to avoid effects such as convection and heat conduction. At the beginning of our experiment, both the ball and the covering should have the same temperature  $T$ . We know from experience that a black body emits more radiation than a light coloured one. We therefore expect the black half of the ball to cool off faster than the polished half. This means that there would be a difference in temperature between the black and the polished parts after a certain period of time. Therefore energy can be drawn out of both parts by attaching a thermo couple, for example, although no energy was introduced. However, this kind of "perpetuum mobile" is not possible and we will have to examine the situation more closely.



**Fig. 2: Kirchoff's Law**

The ball is also exposed to the radiation field of the covering which radiates at an irradiance of  $L_{es}$ . The polished part radiates at an irradiance of  $L_{e1}$  and absorbs radiation from

the covering at a degree  $\alpha_1$ . The black ball radiates at  $L_{e2}$  and absorbs radiation from the covering at a degree  $\alpha_2$ . The initial temperature  $T$  can only be maintained if the black part of the ball absorbs more radiation from the covering than the polished half. The following equation is applicable in this case

$$L_{e1} = \alpha_1 \cdot L_{es} \text{ and } L_{e2} = \alpha_2 \cdot L_{es} \text{ or} \tag{1.3.2}$$

$$\frac{L_{e1}}{\alpha_1} = \frac{L_{e2}}{\alpha_2} = L_{es} \tag{1.3.3}$$

It can be easily concluded that the same relationship applies for areas on the ball which have other values for  $L_e$  and  $\alpha$ . This means that the relationship of the irradiance  $L_e$  of any given thermal radiator to its degree of absorption  $\alpha$ , at the same temperature, is a constant  $L_{es}$ . This constant is physically important. In the limiting case, a black body can only have the degree of absorption  $\alpha = 1$ . This would then be a case of an absolute black body which absorbs all radiation.

We have presumed this in our experiment for the covering shown in Fig.2, since the degree of absorption on the right hand side of Eq.1.3.3 is identical with 1. It therefore applies to any temperature.

**The ratio of the irradiance ( $L_e$ ) over the degree of absorption ( $\alpha$ ) of any given thermal radiator is equal to the irradiance ( $L_{es}$ ) of the black body at the same temperature.**

$$\frac{L_e(T)}{L_e(T)_s} = \alpha(T) \tag{1.3.4}$$

Or, to put it in another way:

The degree of emission in any given thermal radiator is the same as the degree of absorption. In the case of the absolute black body with the degree of absorption  $\alpha = 1$  the degree of emission is  $\varepsilon = 1$ .

We have not as yet considered the wavelength of the radiation. Kirchoff's great achievement was, that he proved that the degree of absorption is the same as the degree of emission at a certain temperature  $T$ , and also that this relationship does not depend on the wavelength  $\lambda$  of the radiation. Kirchoff proved that

$$\int_0^\infty \varepsilon(\lambda, T) d\lambda = \int_0^\infty \alpha(\lambda, T) d\lambda \tag{1.3.5}$$

or

$$\int_0^\infty [\varepsilon(\lambda, T) - \alpha(\lambda, T)] d\lambda = 0 \tag{1.3.6}$$

The following applies for every temperature and wavelength:

$$\varepsilon(\lambda, T) = \alpha(\lambda, T) \tag{1.3.7}$$

The importance of this is stated in the following:

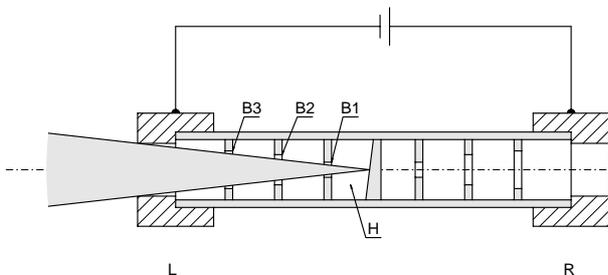
**The radiation of any given thermal radiator is dependent on its absorption capacity, but not on any of its other specific qualities.**

Thanks to Kirchhoff's work the expression we were looking for (1.3.1) can now be simplified to

$$E(\lambda, T) = ? \quad (1.3.8)$$

Kirchhoff has also given details in his work done around 1861, on how to construct an ideal black body radiator, which absorbs radiation at all wavelengths and therefore has the highest possible degree of emission of a thermal radiator. He says:

**"If a space is filled with bodies of the same temperature and no radiation can penetrate these bodies, the quality and intensity of the radiation cluster within the space is such that it seems to originate in a perfect black body of the same temperature and therefore does not depend on the constitution and form of the bodies, but only on the temperature."**



**Fig.3: Black body radiator**

Fig.3 shows the set up of a black body radiator. A tube made of tungsten, for instance, has a cavity H, from which radiation is passed out through the diaphragms B1 to B3. The tube itself is fixed into the copper rings R and L through which the electric current for heating the chamber is introduced simultaneously. The temperature of the cavity is determined by suitable means on the right hand side R. Thermo couples can be used according to the temperature or the radiation law can be applied, which we will deduce shortly. As simple as the experimental set up of Fig.3 may seem, the derivation of the formula for spectral energy distribution  $E(\lambda, T)$  was the starting point for quantum mechanics, a new kind of physics, which caused great excitement at the beginning of this century. At that time, people had tried to find out the spectral distribution of light energy emitted by a black body theoretically. Lord Rayleigh (Philos.Mag.Vol.49 (1900) P.539) and J.H. Jeans (Philos.Mag.Vol.10 (1905) P.91) used a simple model for the absorption and emission mechanism as a basis, which stemmed from the laws of statistics of the gas theory and other thermodynamic disciplines applicable till then. Both authors calculated that the intensity of the radiation emitted by the black body radiator was proportional to the square of the frequency of the radiated light. According to the most elementary experiences, however, the intensity at high frequencies (UV light) is close to zero. All attempts to eliminate this contradiction using classical physics (there

was no other kind at the time) failed. It was an absolute catastrophe. Try and imagine what physicists must have felt at that time. After thinking that physics was a complete science and nature could be interpreted without any problems, this was a shock that caused many doubts.

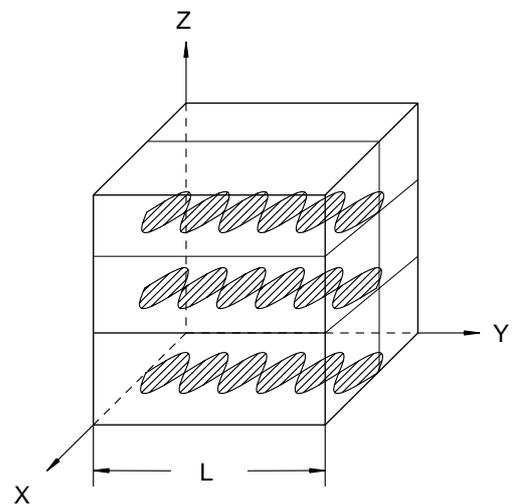
### 1.3.1 Radiation law $u(\lambda, T)$

Max Planck (Verh.dtsch.physik.Ges.(14.Dec.1900) and Ann.Physik(4)Vol.4(1901)P.553-564) found a solution to this UV catastrophe (still called so today) with a totally new hypothesis according to which the absorption and emission processes should be carried out in an elementary way. The strange relationship, where the energy in such an elementary process is proportional to the light frequency is still applicable.

$$E = h \nu \quad (1.3.9)$$

The proportional factor  $h$  is known as Planck's constant or the quantum of action. Planck was able to formulate a radiation law with the help of this hypothesis which has proved successful till today. Planck determined this constant,  $h = 6.57 \cdot 10^{-34} \text{ W s}^2$  through comparative measurements. Today's value is  $6.626196 \cdot 10^{-34} \text{ W s}^2$ . Planck laid the foundations for quantum physics with his hypothesis. As we know from history, it took a long time for classical physicists to admit that classical physics is only marginal to quantum physics.

Let us now return to the derivation of the function  $E(\lambda, T)$ , which was mainly influenced by Rayleigh and Jeans but finally formulated by Planck. We will imagine the full radiator to be a cube with the edge length  $L$ .



**Fig.4: Cavity resonator with standing light waves**

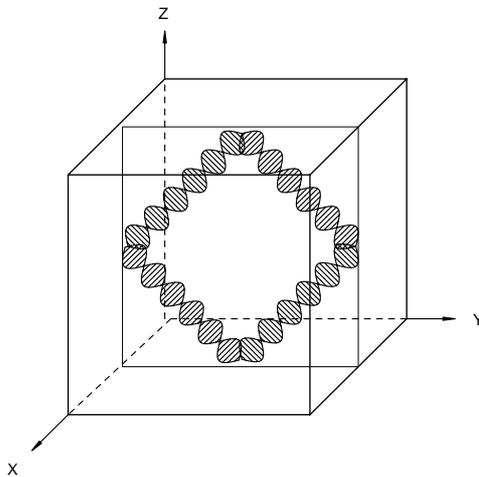
We found out in Chapter 1.3 (Fig.2) that the inner walls of the black body radiate and absorb. There is obviously a radiation field within the box which consists of standing waves since there is a reverse radiation to the wall and from the wall. We must first try and answer the question, how many standing light waves of a particular wavelength  $\lambda$  are present in a box with edge length  $L$ . We will carry out the same procedure for every possible wavelength. The possible number of standing waves in each case will give us

the answer to the question, how much energy does the particular number of standing waves have. This type of box has been shown in Fig.4.

Only standing waves that can fit into the box are formed, i.e. the distance between the nodes must be integral to the length L. Since the distance between the nodes is just about half that of the wavelengths, the condition

$$L = n \frac{\lambda}{2} \quad (1.3.10)$$

must be fulfilled. In this case n is a whole number and  $\lambda$  is the wavelength of light radiation. In Fig.4 only the waves formed in the direction of the Y-axis are shown. The same is applicable to the Z or X axis. We must also take the possibility of a three dimensional standing wave, as shown in Fig.5, into consideration.



**Fig.5: Spatial standing wave propagating in the ZY-plane**

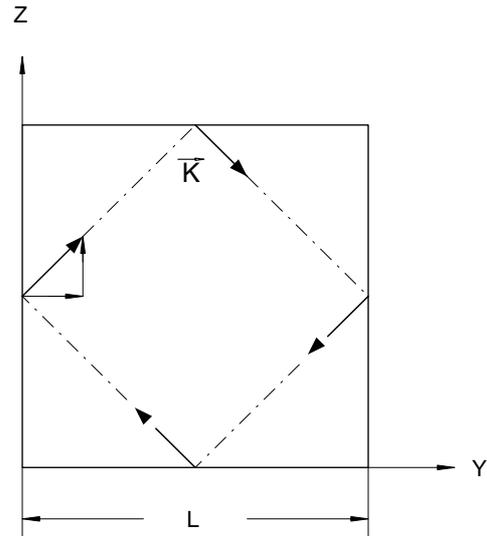
At this point, if not before, the wave vector must be introduced:

$$\vec{K} = \vec{e} \cdot \frac{2 \cdot \pi}{\lambda} \quad (1.3.11)$$

The vector  $\vec{K}$  points in the direction of propagation of the light wave.  $\vec{e} = (e_x, e_y, e_z)$  is its directional vector where  $|\vec{e}| = 1$ . The amount of  $\vec{K}$  is also called the wave number:

$$|\vec{K}| = \frac{2 \cdot \pi}{\lambda} \quad (1.3.12)$$

The dimension of the wave number is  $m^{-1}$  and for practical reasons spectroscopists would prefer this to be in  $cm^{-1}$ . The propagation of light can be shown and described in an easier way because of the introduction of the wave vector. This is clarified in Fig.6, which shows the situation of Fig.5 using wave vectors.



**Fig.6: Like Fig. 5 but representation with wave vectors**

According to Eq. 1.3.10 the condition  $L = n\lambda/2$  must be fulfilled if standing waves are to be formed. We can see in Fig.6 that the wave vector is represented by its orthogonal components. Therefore the following condition is applicable for the formation of standing waves in any given direction in space:

$$\vec{K} = (n_1, n_2, n_3) \cdot \frac{\pi}{L} \quad (1.3.13)$$

and

$$|\vec{K}| = K = \frac{\pi}{L} \cdot \sqrt{(n_1^2 + n_2^2 + n_3^2)} \quad (1.3.14)$$

If we now use the relationship between the wavelength  $\lambda$ , the frequency  $\nu$  and the speed  $c$  of light waves

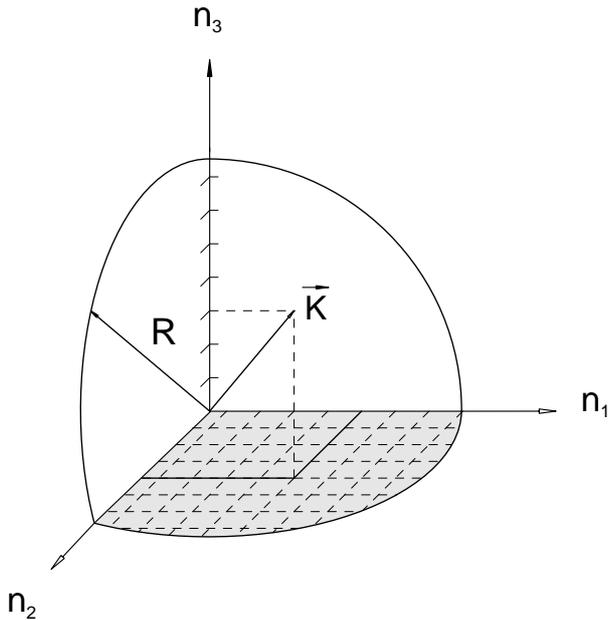
$$\nu = \frac{c}{\lambda} \quad \text{or with} \quad \omega = 2\pi \cdot \nu \quad (1.3.15)$$

the frequencies of the standing waves can also be determined by:

$$\omega = \frac{\pi \cdot c}{L} \cdot \sqrt{(n_1^2 + n_2^2 + n_3^2)} \quad (1.3.16)$$

What has been achieved till now? We know that standing light waves - or to put it differently - a stationary radiation field, can be formed in a cubic box (resonant chamber). This involves discrete, integral values n, which show how many oscillation nodes are formed at a given length L of the cavity and at a particular wavelength or frequency. Every n or  $n_i$  fulfilling Eq.1.3.16 marks a particular state of the radiation field. This state defined by n is called the oscillation mode or simply "mode". The answer to the question, how many such modes exist in a particular frequency interval provides the spectral energy distribution of the emission of a black body radiator. But first we have to find out the number of possible modes. This is done simply by counting them. We will use a trick for this

purpose that is often used in statistical physics and which is shown in Fig.7.



**Fig.7: Representation within the wave vector coordinate system**

If we remember the equation with a sphere with radius R

$$R = \sqrt{x^2 + y^2 + z^2} \quad (1.3.17)$$

and write Eq.1.3.14 in the form

$$\frac{K \cdot L}{\pi} = \sqrt{n_1^2 + n_2^2 + n_3^2} \quad (1.3.18)$$

then compare it to Eq.1.3.17 the result will be an analogous equation. We can take Eq.1.3.18 as an equation for a sphere. The radius of this sphere is  $K L/\pi$  or  $2L/\lambda$  and  $n_1 n_2 n_3$  represent the x y z coordinates. Because the n's are integral and positive they can produce only an eighth of the full sphere, formed from a three dimensional lattice with the lattice constant 1. Therefore only those wave vectors  $\vec{K}$  whose components coincide with the n values are permissible, or to put it differently, every intersecting point of the lattice produces a valid solution for a wave vector  $\vec{K}$  of a standing wave. The answer to the question posed at the beginning, regarding the number of modes required for a given length L of the resonant cavity can, in fact, be found after simply counting the intersecting points of the lattice. This can also be done analytically, using the formula for the volume of a sphere.

$$V_{sphere} = \frac{4}{3} \cdot \pi \cdot R^3 \quad (1.3.19)$$

For an eighth of a sphere with a "radius" for an upper frequency limit at  $\omega_{max}$ .

$$N(\omega_{max}) = \frac{1}{8} \cdot \frac{4}{3} \pi \cdot \left( \frac{L \cdot \omega}{\pi \cdot c} \right)^3 \quad (1.3.20)$$

Since vertically polarised light is not mutually influencing the number N of modes increases by a factor of 2 and we obtain the following solution:

$$N(\omega_{max}) = \frac{\pi}{3} \cdot \left( \frac{L \cdot \omega}{\pi \cdot c} \right)^3 \quad (1.3.21)$$

If we divide Eq.1.3.21 by the volume  $L^3$  of the resonant cavity, we obtain the mode density  $n(\omega_{max})$

$$n(\omega_{max}) = \frac{\pi}{3} \cdot \left( \frac{\omega}{\pi \cdot c} \right)^3 = \frac{8\pi}{3} \left( \frac{\nu}{c} \right)^3 \quad (1.3.22)$$

By differentiating after  $d\nu$ , we get the mode density in a frequency interval  $d\nu$

$$dn(\nu_{max}) = \frac{8\pi}{c^3} \cdot \nu^2 d\nu \quad (1.3.23)$$

Up to this point, calculations were made according to Rayleigh. Rayleigh then followed the laws of the kinetic gas theory to obtain the spectral energy. When using this law the energy  $E_m = 1/2kT$  is attributed to every degree of freedom of a system in equilibrium. In this case k is the Boltzmann constant and T the absolute temperature. The average kinetic energy in harmonic oscillations is equal to the average potential energy. Therefore the energy per degree of freedom for harmonic oscillation processes is  $kT$ . Rayleigh's result, the so called UV catastrophe, was as follows:

$$u(\nu, T) = dn(\nu_{max}) \cdot k \cdot T$$

$$u(\nu, T) = \frac{8\pi}{c^3} \nu^2 \cdot k \cdot T \cdot d\nu \quad (1.3.24)$$

Of course, Rayleigh knew that his results contradicted the experiments, but they corresponded perfectly to the laws of physics known up to that time. The starting point for these laws is the Boltzmann distribution.

### 1.3.2 The Stefan-Boltzmann Laws

Ludwig Boltzmann lived from 1844-1906 and is the founder of the atomic theory and electrodynamics. In 1879 J. Stefan deduced, from measurements taken by French physicists, that the density of energy for all frequencies of a resonant cavity is the same and equal to a constant multiplied by the temperature to the power four.

$$E(T) = k \cdot T^4 \quad (1.3.25)$$

Stefan presumed that this law applied to all radiators. In 1884 Boltzmann limited it to black bodies and finally derived and formulated it. It has been confirmed by measurements and its fundamental importance lies in the proof that a volume filled with radiation can be treated as if there were a gas in the volume. Boltzmann got his results

by using statistical methods. Just as a probability is calculated by cubing a certain number, Boltzmann solved the problem of how many particles of a gas dissolve out of the total quantity  $N$  at a temperature  $T$  within an energy interval  $dE$ . His result was the important Maxwell-Boltzmann distribution:

$$\frac{dn}{N} = \frac{\sqrt{8/\pi}}{\sqrt[3]{k^2 T^2}} \sqrt{E} \cdot e^{-\frac{E}{kT}} \cdot dE \quad (1.3.26)$$

Boltzmann obtained as average energy for all three degrees of freedom of a gas molecule using Eq.1.3.26.  
 $E_m = 3/2 k T$  (1.3.27)

and thus found the  $T^4$  law (The Stefan-Boltzmann Law) which was also correct for the resonant cavity. Lord Rayleigh used these findings, which were proved to be right. But this still led to the UV catastrophe, which Planck solved with his new hypothesis and which showed that Rayleigh was right, but only under certain conditions.

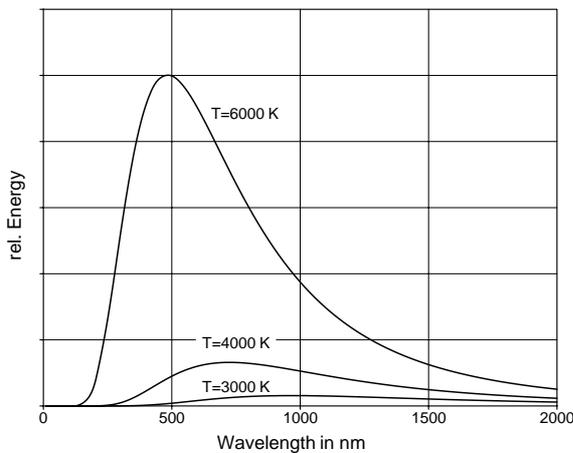
### 1.3.3 Planck's Radiation Law

Max Planck carried out the classical calculations once more, but this time he intuitively understood that only energies that fulfilled the condition

$$E = n h \nu \quad (1.3.28)$$

were permissible. Without being aware of the future consequences, i.e. the development of a new kind of physics called quantum physics, Planck maintained that the radiation energy was not distributed continuously, as physicists had correctly assumed till that point, but in an integral quanta  $n$ . However, a constant  $h$  was necessary for the required dimension of energy on the right hand side of Eq.1.3.28. Boldly assuming this, he calculated that the average energy  $E_m$  was no more  $kT$  as Rayleigh had assumed based on Boltzmann's work, but

$$E_m = \frac{h \cdot \nu}{e^{\frac{h \cdot \nu}{k \cdot T}} - 1} \quad (1.3.29)$$



**Fig 8: Planck's law of radiation for different temperatures of the black body radiator**

If this is multiplied with (1.3.23) the energy density  $u(\nu,T)$ ,

analogous to the way Rayleigh proceeded, would now result in

$$u(\nu, T) = \frac{8\pi}{c^3} \cdot \frac{h \cdot \nu^3}{e^{\frac{h \cdot \nu}{k \cdot T}} - 1} \cdot d\nu \quad (1.3.30)$$

With  $\nu = c/\lambda$  Eq.1.1.30 is a function of the wavelength

$$u(\lambda, T) = \frac{8\pi \cdot c}{\lambda^5} \cdot \frac{h}{e^{\frac{h \cdot c}{\lambda \cdot k \cdot T}} - 1} \cdot d\lambda \quad (1.3.31)$$

This is the famous Planck Radiation Formula, and we have now completed the derivation of the energy of a full thermal radiator as a function of its temperature and the distribution of energy on wavelengths emitted by the radiator. In Fig.8 this function has been calculated and graphically represented for three temperatures. If we analyse the dimensions of  $u(\lambda,T)$  we will get  $Ws/m^3$  or energy per unit volume of the black radiator. However this variable poses problems in radiation physics since the volume of the black body radiator cannot be measured immediately. The energy per unit of volume is transported at a speed  $c$  and distributed over the entire solid angle  $\Omega = 4 \pi \Omega_0$ .  $\Omega_0$  is the unit of the solid angle. (see point 1.5.4) We must now define

$$u(\lambda, T) \cdot c = 4\pi \cdot \Omega_0 \cdot L_e(\lambda, T) \quad (1.3.32)$$

$$L_e(\lambda, T) = \frac{c}{4\pi \cdot \Omega_0} \cdot u(\lambda, T)$$

$L_e(\lambda,T)$  has the dimension  $Wm^{-2} sr^{-1}$  and is known as radiance.  $sr$  is the steradian and is defined under 1.5.4.

Let us now return to Planck's radiation law, which will now be written in units of radiance.

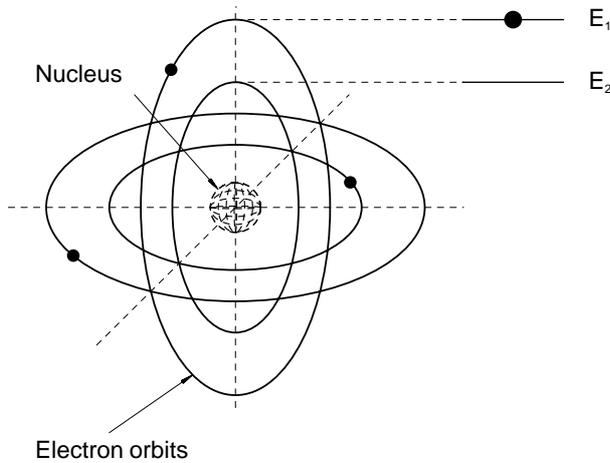
$$L_{e\lambda}(\lambda, T) = \frac{2 \cdot c^2}{\Omega_0 \cdot \lambda^5} \cdot \frac{h}{e^{\frac{h \cdot c}{\lambda \cdot k \cdot T}} - 1} \quad (1.3.33)$$

$L_{e\lambda}(\lambda,T)$  is the radiance per wavelength interval  $d\lambda$  and is therefore called spectral radiance. It is only through the integration over all  $\lambda$  that radiance is formed. The connection between radiance and luminance that is so important in photometry will be explained in Chapter 1.5.1.

### 1.4 Cold Light

We have learned about one kind of light production and have also described the radiation of bodies at a temperature  $T$ . At the same time as these examinations and calculations were being carried out, spectroscopists were working on the meaning of light emission and the absorption of atoms and molecules which, contrary to the continuous radiation of the thermal radiator showed a structure that was characteristic for certain atoms and molecules. Apart from its function as a source of light, physicists were primarily interested in the meaning of spectra, since they could find out more on the

structure of atoms and molecules in this way. After the research into the physics of atoms also came to standstill, Niels Bohr took up Max Planck's hypothesis.



**Fig.9: Bohr's Atom**

He expounded his existing theory on the atom model and said that the light emitted or absorbed by atoms could only have an energy of  $E_2 - E_1 = h\nu$  (Fig.9). The work and measurements had proved that discrete energy must be anticipated for both resonant cavities and appearances of atomic emissions. Einstein began looking for a single description for both these sources of light. He was able to solve this problem in 1917 when he derived Planck's hypothesis once more in his own way. He thought of a way of combining both light sources. Instead of a sphere, like the one we used in Fig.2, he put an ensemble of "Bohr" atoms in the resonant cavity at a temperature T. In thermal equilibrium  $E(\nu, T)$  will be an energy distribution which must be marked by the qualities of the atoms as a "cold light source". The new distribution  $E(\nu, T)$  must be found. The first step is to examine the atom ensemble, which we presume has only two energy levels, as shown in Fig.9. Since the atoms are in a radiation field, they can take up or absorb energy. The absorption is connected to an emission. If we call the number of electrons in state 1,  $n_1$ , the temporal change in this quantity will be

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u(\nu) \quad (1.4.1)$$

In this case  $u(\nu)$  is the density of energy at the frequency at which the transition from state 1 to state 2 is resonant, i.e. it is the frequency at which  $E_2 - E_1 = h\nu$  is fulfilled. This frequency is called the resonant frequency. It is evident that the temporal change from  $dn_1/dt$  is dependent on the number  $n_1$  itself, on one hand, and on the density of energy of the radiation with the frequency  $\nu$ , on the other. A constant  $B_{12}$  is necessary for a correct equation in terms of dimension. The minus sign is required because the number of electrons in state 1 decreases through the absorption. The same observation is carried out for state 2. We will call the number of electrons in this state  $n_2$ . The electrons return to state 1 from state 2 whilst emitting radiation. The transition from 2 to 1 is released (induced) by the existing radiation field of the resonator and takes also place coincidentally

(spontaneously). So, two types of emission are responsible for depopulating state 2, the induced and the spontaneous emission. The temporal change in the number  $n_2$  is

$$\frac{dn_2}{dt} = -B_{21} \cdot n_2 \cdot u(\nu) - A_{21} \cdot n_2 \quad (1.4.2)$$

Nothing has been left out of the last term since the spontaneous emission does not depend on the surrounding radiation field and is of a statistical nature. It takes place even when there is no radiation field. Until the principles of quantum mechanics were defined by Heisenberg and Schroedinger, it was accepted that spontaneous emission was similar to radioactive decay, in that it could not be influenced from the outside. Quantum electrodynamics has shown that a spontaneous emission is an emission induced by zero point energy. So as not to take this too far at this point, the following must be noted with reference to zero point energy.

In the cavity there is an average field energy of at least  $E_0 = 1/2 h\nu$ . The spontaneous emission is triggered off by this energy.

Let us go back to our resonant cavity-two level atom system. In stationary equilibrium, the same number of electrons must go from state 1 to 2 (with a photon being absorbed from the radiation field) and vice-versa (emission of a photon into the radiation field)

$$\frac{dn_1}{dt} = \frac{dn_2}{dt} \quad (1.4.3)$$

or

$$B_{12} \cdot n_1 \cdot u(\nu) = B_{21} \cdot n_2 \cdot u(\nu) + A_{21} \cdot n_2 \quad (1.4.4)$$

The Boltzmann distribution is also valid in the thermal equilibrium for the population numbers of level 1 and level 2

$$n_2 = n_1 \cdot e^{-\frac{E_2 - E_1}{k \cdot T}} \quad \text{or} \quad n_2 = n_1 \cdot e^{-\frac{h \cdot \nu}{k \cdot T}} \quad (1.4.5)$$

By substituting (1.4.5) in (1.4.4) we get

$$u(\nu, T) = \frac{A_{21}}{B_{12}} \cdot \frac{1}{e^{\frac{h \cdot \nu}{k \cdot T}} - B_{21} / B_{12}} \quad (1.4.6)$$

Since Planck's law must be valid also in equilibrium we get by comparison of (1.3.33) with (1.4.6) the significant Einstein coefficients

$$B_{12} = B_{21} \quad \text{und} \quad \frac{A_{21}}{B_{12}} = 8\pi \cdot \frac{h \cdot \nu^3}{c^3} \quad (1.4.7)$$

### 1.4.1 Natural line width

Let's look on Eq. (1.4.1):

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u(\nu)$$

$B_{12}$  can be considered as the probability for a transition from level 1 to level 2 by absorption.

This is also analogous to the coefficient  $B_{21}$ , which however indicates the probability of the reverse process, i.e. the emission.

The coefficient for the spontaneous emission  $A_{21}$  gives us another interesting piece of information on the system, which is easy to find.

Let us take, for example, the process of the spontaneous emission by itself.

$$\frac{dn_2}{dt} = -A_{21} \cdot n_2 \quad (1.4.8)$$

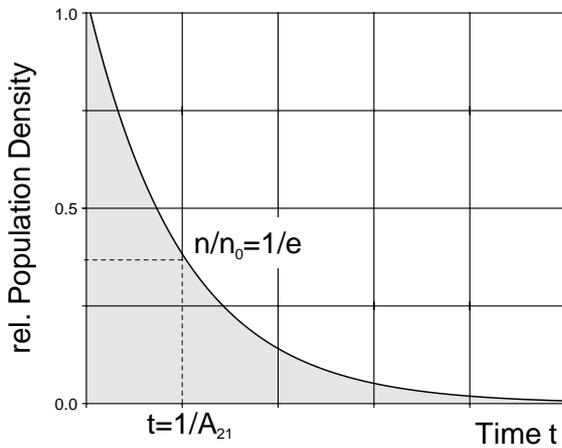
This differential equation can be solved using the additional equation

$$n_2(t) = C \cdot e^{-\alpha \cdot t} \quad (1.4.9)$$

$\alpha = A_{21}$  can be found by comparing the two and the solution will then be

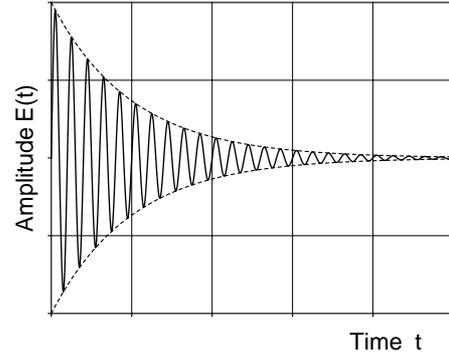
$$n_2(t) = n_2(t=0) \cdot e^{-A_{21} \cdot t} \quad (1.4.10)$$

Fig.10 shows this function graphically. This curve and  $A_{21}$  can be determined experimentally. The time  $t$  which  $n_2(t)$  took to gain the value  $n_2(t=0) e^{-1}$  must be deduced. The result will then be  $t = 1/A_{21}$ . Obviously the reciprocal value of the Einstein coefficient  $A_{21}$  represents a suitable definition for the "life time  $\tau$  of a state"



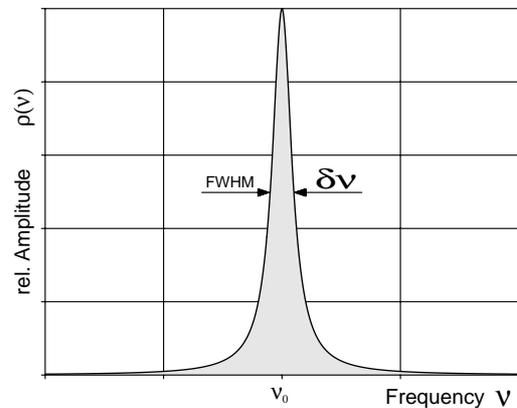
**Fig.10: Decay curve of a state**

More information can be obtained from the disintegration curve. Photons or a radiation field are produced because of the transition from state 2 to 1. However, the intensity of the radiation decreases exponentially with the time (Fig.11). In view of the preceding findings, the frequency of the radiation should be fixed to  $E_2 - E_1 = h \nu_0$ .



**Fig.11: Spontaneous emission as a damped oscillation**

A power spectrum of the spontaneous emission is obtained using a Fourier analysis for non-periodic processes, which has the main frequency  $\nu_0$  apart from other frequency parts. The result of this kind of Fourier transformation is illustrated in Fig.12



**Fig.12: The Fourier transformation of a damped oscillation as observed for spontaneous emission. It consists of the transition frequency  $\nu_0$  and a complete spectrum described by a Lorentz function.**

The Fourier transformation of the damped oscillation gives the following result:

$$\rho(\nu) = \frac{1}{4\pi^2 \cdot (\nu - \nu_{21})^2 + (1/2 \cdot \tau_s)^2} \quad (1.4.11)$$

This type of curve represents a Lorentz curve.  $\nu_{21}$  (also called  $\nu_0$ ) is the resonant frequency and  $\tau_s = \frac{1}{A_{21}}$  the

average life time of state 2. The FWHM (Full Width at Half Maximum) of the curve as shown in Fig.12 is calculated by inserting the value of  $\rho(\nu) = 1/2$ . The result is:

$$\delta(\nu)_{nat} = \frac{1}{2\pi} \cdot A_{21} \quad (1.4.12)$$

which is the natural line width of a transition, defined by the Einstein coefficient  $A_{21}$  which has a particular value for every transition. The results obtained can also be interpreted as if the state 2 did not have any clearly defined energy, but a broadening with half-width  $\Delta E = 2 \pi h A_{21}$ . This means

that the state is somewhat blurred. Quantum mechanics has shown this effect to be extremely important. It is known as the Heisenberg uncertainty principle, after the person who found it. In the case of normal optical transitions the value of  $\tau_s$  lies between  $10^{-8}$  to  $10^{-9}$  seconds. This life time, determined by spontaneous transitions alone, is crucial for the so called **natural width** of a spectral line. To clarify the ways in which we term things, we must emphasise briefly at this point, that there is a difference between the width of a state and the width of a line, as well as between the terms state and line. There are always states for atoms and it is never stated whether the state is occupied or empty. A line is only formed if an emission is caused by the transition from, for example, state 2 to 1. The line is a word commonly used by spectroscopists. They use their spectroscopes to produce photographic plates, for example, on which fluorescent light is shown according to its wavelengths. The use of slits in the optical beam path makes it easier to evaluate the spectra. A line spectrum of this kind is shown in Fig.13.

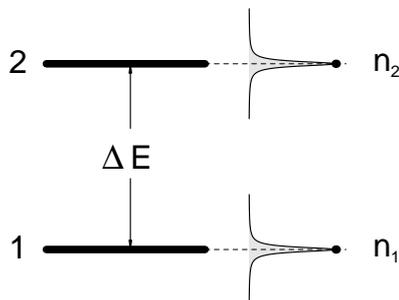


**Fig.13: Recording the emission of a light source with corresponding energy levels results in a line spectrum**

Apart from the emission wavelengths, the spectrum in Fig.13 shows the line widths. It must be observed, in this case, that the measuring apparatus makes the line widths seem wider than they actually are. Naturally, it was the aim of spectroscopists to create instruments which could give the closest reading of the actual line widths.

**1.4.2 Line broadening**

According to the preceding account, transitions are only possible if the energy of the absorbed or emitted photon lies within the natural width of the transition.



**Fig.14: “Uncertainty” of energy levels because of the natural line width**

Actually, nature is not so critical and we can observe that even photons take part in processes which deviate slightly in energy. This is because the energy levels are broadened in addition to the natural line width, due to various mechanisms. The natural line width is actually very small compared to other broadening effects. There is a broadening  $\delta E$

which depends on the temperature of the atoms and how they are influenced by the interaction with their environment. This leads to the acceptance of photons within this area. The width of the transition is indicated by its FWHM in each case. All of this also applies to the emission.

**1.4.3 Homogeneous line broadening**

A line is homogeneously broadened when all the atoms or molecules have the same characteristics and all of them interact with their environment in the same way. The natural broadening is a homogeneous broadening, since it is the same for all atoms and molecules in an ensemble. Homogeneous broadening can be found in solids with regular crystal structure in which the atoms considered are in equivalent lattice sites. The interactions with the crystal lattice lead to a broadening of the states that is far beyond the natural width, but which is homogeneous when the lattice sites are symmetrical and of equal value. Gases are known for their inhomogeneous broadening and this will be discussed in the next section. In this case the absorption and emission lines are no more homogeneously but inhomogeneously broadened.

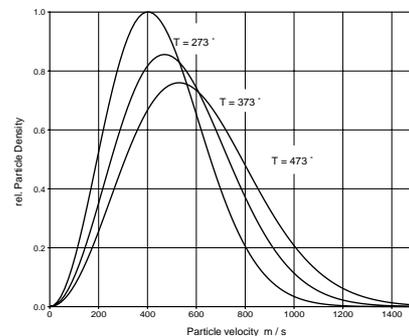
**1.4.4 Inhomogeneous line broadening**

Materials in which atoms and molecules have different qualities do not behave in the same way as solids. This is particularly obvious in gases where atoms or molecules move more or less freely at varying speeds  $v$ . The number of particles  $n(v)$  in a total quantity of  $N$  with a mass  $m$  within a speed interval of  $v$  to  $v+dv$  is described by the Maxwell-Boltzmann distribution

$$\frac{n(v)}{N} = \frac{4v^2}{\sqrt{\pi}} \cdot \frac{1}{\sqrt[3]{(2kT/m)^2}} \cdot e^{-\frac{mv^2}{kT}} dv \quad (1.4.13)$$

The above equation applies to all directions in space. To make things simple, we will take into consideration only one direction, e.g. the direction  $x$  :

$$\frac{n(v_x)}{N} = \sqrt{\frac{2 \cdot k \cdot T}{m}} \cdot e^{-\frac{m \cdot v_x^2}{k \cdot T}} \cdot dv_x \quad (1.4.14)$$



**Fig.15: Distribution of N<sub>2</sub> molecules according to their speed**

**1.4.4.1 Doppler broadening**

The passive observer will now see a shift in the absorption

or emission frequency due to the speed of the particles. The value is

$$v = v_0 (1 \pm v/c) \quad (v \ll c),$$

$v_0$  is the absorption or emission frequency of the resting particle and  $c$  is the speed of light. This phenomenon has been named the Doppler effect after its discoverer (Ch. Doppler, Abh. D.K. Boehmischen Ges. d. Wiss. (5), Vol. II (1842) P.465).

If Doppler's equation is inserted in the equation for the distribution of speed  $n(v_x)$ , the line broadening produced by the movement of particles is obtained. Since the intensity is proportional to the number of emitting particles the distribution of intensity will be

$$I(v) = I(v_0) \cdot e^{-\frac{c^2}{v_w^2} \cdot \frac{(v-v_0)^2}{v_0^2}} \quad (1.4.15)$$

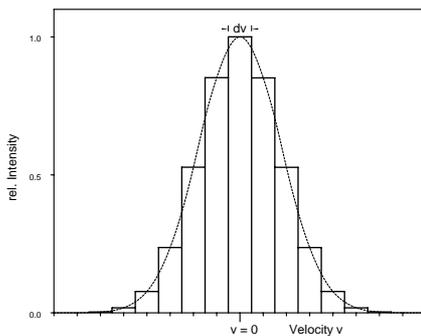
$v_w$  is the most probable speed in this case

$$v_w = \sqrt{\frac{2 \cdot k \cdot T}{m}} \quad (1.4.16)$$

The calculation for the FWHM is done in the same way as with the natural line width and the result is

$$\Delta v_{doppler} = \sqrt{4 \cdot \ln 2} \cdot \frac{v_0}{c} \cdot \sqrt{\frac{2 \cdot k \cdot T}{m}} \quad (1.4.17)$$

This result shows us that the line broadening is increasing due to the Doppler effect when there are higher frequencies  $v_0$  or smaller wavelengths (IR) and higher temperatures. The line broadening is smaller when the mass  $m$  of the particles increases. The line profile corresponds to a Gaussian distribution (Fig. 16).



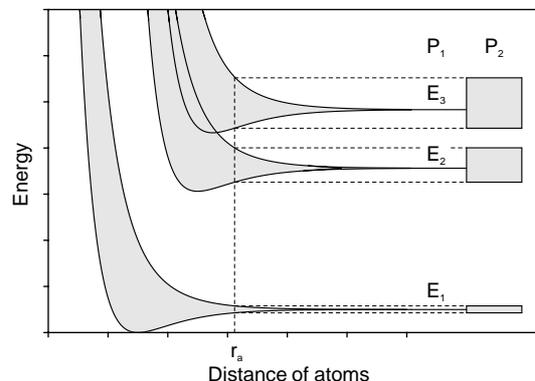
**Fig.16: Inhomogeneous line profile with speed intervals  $dv$**

The histogram is close to the smooth curve when the speed intervals  $dv$  are small. You may wonder what the purpose of these speed intervals  $dv$  or frequency intervals  $dv$  is. Strictly speaking this has formal reasons. All the results given till now are based on statistical statements. Reducing an interval to zero would be like talking about taking out a particle with an exact speed or frequency from an ensemble. It is almost impossible for this to succeed. So, the concept of intervals was introduced to describe the intervals on the basis of definite probabilities. If this is achieved the

intervals can be made smaller and we can work with „smooth curves“. Thanks to Einstein's work we now have a common description for thermal and cold radiators. Both these types of "lamps" are indispensable in modern technology. The incandescent lamp, which provides light to us daily is a thermal radiator. Technically, cold radiators such as fluorescent or arc lamps are some of the most important light sources with which high luminous powers are produced to light up large areas. We must not fail to mention the laser, another important cold radiator, which, however generally does not count as a light source because of its special characteristics. In actual fact, a cold radiator emits a line spectrum with a comparatively narrow band width, consisting of a number of lines. Radiators with an emission suited to the sensitivity of the human eye (Fig.1) are wanted for lighting purposes. This is why the pressure in this type of lamp is increased. The lines are then broadened and the desired emission spectrum is adjusted. The effect causing line broadening due to increased pressure is known as pressure broadening and is explained in the next chapter.

**1.4.4.2 Pressure broadening**

Let us first observe the situation where the atoms are wide apart from each other. This is the case at pressure  $P_1$ . As shown in chapter 1.4.1 the emission spectrum consists of narrow lines superimposed by the unavoidable Doppler broadening. If we increase the pressure to  $P_2$ , the number of atoms per unit volume will increase. Now the atoms are packed more densely together. Their energy states will change at this point, due to the interaction of their electrons. The average distance  $r_a$  of the atoms depends on pressure and temperature of the gas used. Unlike regular solids, e.g. crystals, the value of  $r_a$  is not discrete but statistically distributed. Several transitions between energy levels are therefore possible and the original line spectrum becomes a band spectrum. If the pressure is increased even further,  $r_a$  becomes smaller and the width of the emission bands increases.



**Fig.17: Line broadening by atom approach caused by increasing the pressure from  $P_1$  to  $P_2$**

Fig.17 shows that in addition to this, the central energy and the main wavelength of the emission band is also displaced. It is then the job of the lamp designer to find suitable atoms

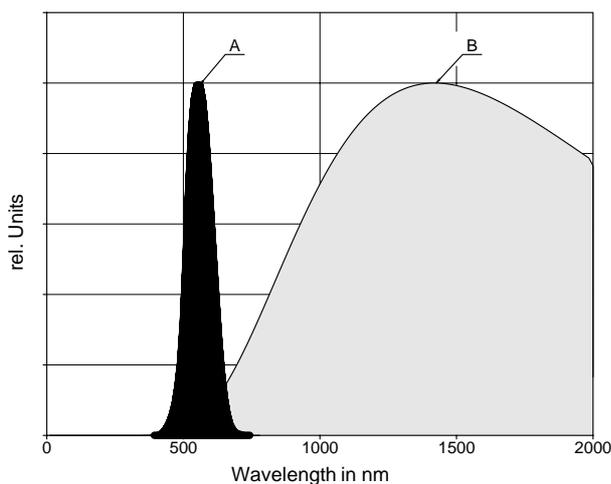
for his lamps, to adjust it to an optimal pressure for a light emission that has a good efficiency on one hand, and is adapted to the spectral sensitivity of the eye on the other. Objective measurement procedures and measuring units must be prepared for the light to judge the progress of the work on lamps. This will be taken up in the following chapter 1.5.

**1.5 Light engineering - Photometry**

Whereas the variables of radiation in the optical area are usually measured in watt seconds or watts, the visible part of optical radiation has its own photometric variables with a basic variable known as candela (cd). It is the sixth variable apart from the basic variables length (m), mass (kg), time (s), intensity of current (A) and temperature (T), which were laid down at the 10th International Conference for Weights and Measures in 1954. All these variables have a standard which is standardised world-wide by calibrating institutes. Take length, for example. For a long time its standard was the primary standard meter in Paris, but it is now defined by the time it takes for light to pass over 1 meter in vacuum. The standards for mass, time etc. are also well known. The standard for the measurement of variables in light engineering, however, are not so well known, since we do not come across them very often in our daily lives. But they are indispensable in the preparation of lighting equipment for the various kinds of places people inhabit.

**1.5.1 Variables in light engineering**

In the previous chapters we deduced the radiation laws for thermal and cold radiators. In the process of doing so, we have taken the whole optical and spectral range into consideration, which covers a wavelength range of 0.2 μm to approx. 1 mm. In light engineering or photometry however, only the range that can be perceived by the human eye is of interest.



**Fig.18: Sensitivity curve of the human eye for day view (curve A) and Planck's energy distribution of a black Platinum radiator at its point of solidification of 2048 K (curve B)**

Wavelength in nm	Sensitivity V(λ)
380	0,0000
390	0,0001
400	0,0004
410	0,0012
420	0,0040
430	0,0116
440	0,023
450	0,038
460	0,060
470	0,091
480	0,139
490	0,208
500	0,323
510	0,503
520	0,710
530	0,862
540	0,954
550	0,995
560	0,995
570	0,952
580	0,870
590	0,757
600	0,631
610	0,503
620	0,381
630	0,265
640	0,175
650	0,107
660	0,061
670	0,032
680	0,017
690	0,0082
700	0,0041
710	0,0021
720	0,00105
730	0,00052
740	0,00025
750	0,00012
760	0,00006
770	0,00003
780	0,000015

**Table 1: Values of the sensitivity curve V(λ) of the human eye for day view**

Since the sensitivity of the eye is a subjective variable, the International Commission for Lighting has laid down the course of the spectral response of the eye V(λ). Several measurements were taken on people to obtain a statistical result for this purpose. Photometry considers the subjective spectral response (sensitivity) of the human eye combined with strict physical rules. The sensitivity of the human eye is the measure of all things in photometry. It has always been a horrendous task for the serious physicist to define a new entity, i.e. current for lighting purposes, by superimposing physical realisations on to a fictitious, physiological

variable. But this definition has remained useful in technology and more is not required anyway. However, work on the serious physical definition of photometric variables is still in process.

**1.5.2 The radiation equivalent  $K_m$**

We can now try to connect the photometric variable  $L_v$  with the radiometric variable  $L_{e\lambda}(\lambda, T)$  in the following form :

$$K_m = \frac{L_v}{\int L_{e\lambda}(\lambda, T) \cdot V(\lambda) \cdot d\lambda} \quad (1.5.1)$$

$K_m$  is a factor of proportionality which controls the calculations of the radiometric variables into photometric variables. We now obtain the photometric variable  $L_v(T)$ , called luminance, analogous to the radiance  $L_{e\lambda}(\lambda, T)$ , defined using Planck's radiation formula for a black body. In table 2 the

relevant variables of radiation physics (radiometry) are shown next to those of light engineering (photometry). We will be paying particular attention to the luminous intensity measured in Candela (cd), which is the sixth basic variable in the internationally standardised system. A black body radiator at a particular temperature is used for the definition of the unit Candela. This temperature should be as high as possible to ensure a high proportion of visible radiation. In addition to this the temperature T must be known. It is of advantage, in this case, to use materials which have a high latent heat. The temperature can thus be kept constant for as long as possible in the phase of transition from the liquid to the solid state. This occurs, for example, in platinum. How to set up an apparatus which defines the Candela will be discussed in Chapter 1.6.1.

**Table 2: Comparison of important variables of radiometry and photometry**

Variables in radiometry			Variables in photometry		
Variable	Symbol	Unit	Variable	Symbol	Unit
Radiant Intensity	$I_e$	$Wsr^{-1}$	Luminous intensity	$I_v$	Candela (cd)
Radiant Power	$\Phi_e$	Watt (W)	Light Flux	$\Phi_v$	cd sr Lumen (lm)
Radiant Energy	$Q_e$	W s	Quantity of Light	$Q_v$	cd sr s lm s
Radiance	$L_e$	$Wsr^{-1}m^{-2}$	Luminance	$L_v$	cd $m^{-2}$
Irradiance	$E_e$	$Wm^{-2}$	Illuminance	$E_v$	Lux (lm $m^{-2}$ )
Radiant Efficiency	$\eta_e$	%	Luminous efficiency	$\eta_v$	lm $W^{-1}$

1.5.3 Calculating  $K_m$

Table 3 shows the values calculated according to Planck's radiation formula of  $L_{e\lambda}(\lambda_i, T=2045^0K)$  and the values of the spectral sensitivity of the eye taken from Table 1. Both sets of values have been put together and the product of both has been formed. Finally  $K_m$  is determined by numerical integration. We often find deviant values for  $K_m$  in other quotations which are probably based on different  $V(\lambda)$  or the integration method used. Here, we must point out that contrary to the classic basic units the standard value found for the Candela cannot be more precise than  $10^{-3}$ .

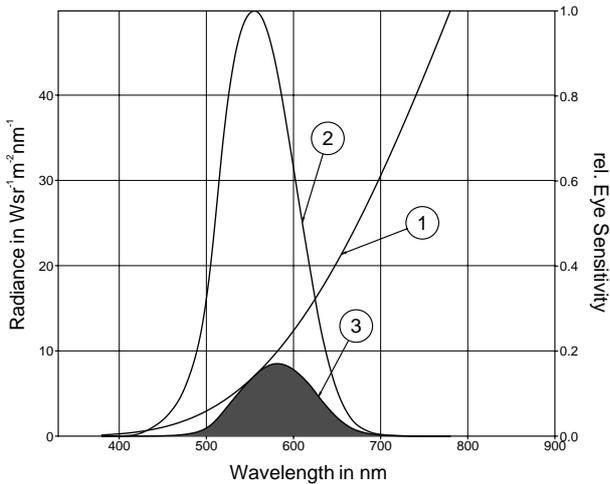


Fig.19: Determination of  $K_m$ , (1) calculated due to Planck's formula at  $T=2045^0K$ , (2) sensitivity curve of the human eye, (3) product of both curves

Table 3: Calculation of  $K_m$

nm	$L_{e\lambda}(\lambda_i, T_{Pt})$ $Wm^{-2}sr^{-1}nm^{-1}$	$V(\lambda_i)$	$L_{e\lambda}(\lambda_i, T_{Pt}) V(\lambda_i)$
380	0,13655302	0,0	0,0
390	0,19279795	0,0001	0,00001928
400	0,26669897	0,0004	0,00010668
410	0,36202784	0,0012	0,00043443
420	0,48292693	0,004	0,00193171
430	0,63386674	0,0116	0,00735285
440	0,81959208	0,023	0,01885062
450	1,0450589	0,038	0,03971224
460	1,3153637	0,06	0,07892182
470	1,6356682	0,091	0,14884581
480	2,01112124	0,139	0,27954585
490	2,44678054	0,208	0,50893035
500	2,94753627	0,323	0,95205422
510	3,51803833	0,503	1,76957328
520	4,16262906	0,71	2,95546663
530	4,88528255	0,862	4,21111356
540	5,68955173	0,954	5,42783235
550	6,57852366	0,995	6,54563104
560	7,55478369	0,995	7,51700977
570	8,62038839	0,952	8,20660974
580	9,77684726	0,87	8,50585712
590	11,02511293	0,757	8,34601049
600	12,36557925	0,631	7,8026805
610	13,79808679	0,503	6,94043765
620	15,32193506	0,381	5,83765726
630	16,93590064	0,265	4,48801367
640	18,63826042	0,175	3,26169557
650	20,42681926	0,107	2,18566966
660	22,29894118	0,061	1,36023541
670	24,25158337	0,032	0,77605067
680	26,2813323	0,017	0,44678265
690	28,38444124	0,0082	0,23275242
700	30,55686855	0,0041	0,12528316
710	32,7943163	0,0021	0,06886806
720	35,09226859	0,00105	0,03684688
730	37,44602918	0,00052	0,01947194
740	39,85075811	0,00025	0,00996269
750	42,30150697	0,00012	0,00507618
760	44,79325256	0,00006	0,0026876
770	47,32092869	0,00003	0,00141963
780	49,87945609	0,000015	0,00074819

$$\sum_i V(\lambda_i) \cdot L_{e\lambda}(\lambda_i, T_{Pt}) \cdot \Delta\lambda = 891,2$$

$$K_m = \frac{6 \cdot 10^5}{\sum_i V(\lambda_i) \cdot L_{e\lambda}(\lambda_i, T_{Pt}) \cdot \Delta\lambda} \equiv 673 \left[ \frac{cd \cdot sr \cdot m^2}{W} \right]$$

1.5.4 The solid angle  $\Omega$

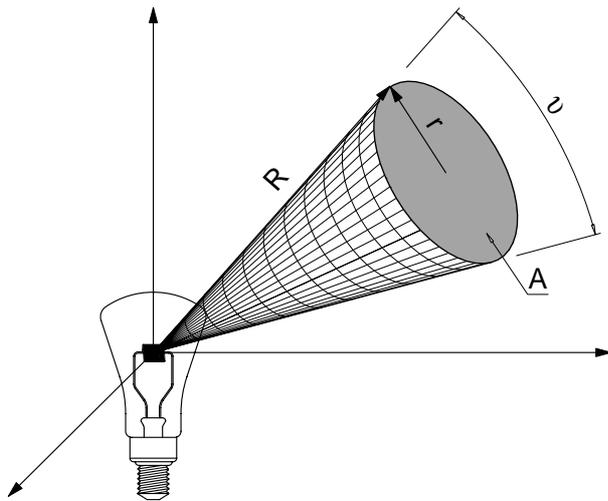


Fig.20: Definition of the solid angle

The frequently used expression, solid angle  $d\Omega$  will be clarified once again with the help of Fig.20. The solid angle  $d\Omega$  is defined as the ratio of the spherical surface  $A$  to the total surface of the sphere of radius  $R$ :

$$d\Omega = \frac{A}{4\pi \cdot R^2}$$

For  $A = 1 \text{ m}^2$  and  $R = 1 \text{ m}$  (unit sphere) we get the unit of the solid angle, the steradian sr

$$1 \text{ sr} \equiv \frac{1}{4\pi}$$

The solid angle 1 sr cuts a cone out of the unit sphere with an angle  $v$  (see Fig.20) If the surface of the corresponding spherical section is approximated by the circular surface  $\pi r^2$  we get with

$$A = \pi r^2$$

for  $A = 1 \text{ m}^2$

$$r = \frac{1}{\sqrt{\pi}}$$

and for  $v$  :

$$\sin\left(\frac{v}{2}\right) = \frac{r}{R} = \frac{1}{\sqrt{\pi}}$$

or  $v \cong 34^\circ$

Fig.20 helps us to understand the individual terms such as luminous intensity, light flux etc. We imagine the radiating body to be dot-shaped. In practice, however, this is seldom the case, although it can be achieved if the presumed radius  $R$  of the sphere surrounding the radiator, is large enough.

The light that passes by the solid angle unit is called radiant intensity  $I_e$  and is measured in  $\text{W/sr}$ . After evaluation with the spectral response of the eye  $V(\lambda)$  we obtain the luminous intensity  $I_v$ , given in Candela. The measurement of the luminous intensity is done either by measuring the radiometric variables followed by calculating the photometric variable, or directly, by comparing it with a standard for luminous intensity (see Chapter 1.6).

1.5.5 Luminous intensity

To measure the radiant intensity in  $\text{Wsr}^{-1}$ , it is necessary to know the solid angle used during the measurement. A diaphragm with a radius  $r$  is used for this purpose and a distance  $R$  to the radiator is selected. The solid angle  $\Omega$  for this arrangement is therefore

$$\Omega = \pi r^2 / 4 \pi R^2 = r^2 / 4R^2$$

1.5.6 Light flux

Let us measure the light flux radiated into the full solid angle. We will have to determine the radiant power  $\phi_e$ . It results from the radiant intensity  $I_e$  by integrating over all solid angles  $d\Omega$  from 0 to  $4\pi$ .

$$\Phi_e = \int_0^{4\pi} I_e(\Omega) d\Omega \tag{1.5.2}$$

After evaluation with the spectral response of the eye  $v(\lambda)$  we get the light flux which is given in units of  $\text{cd sr}$ . In case the radiator radiates uniformly in all directions of the space the radiant power will be

$$\Phi_e = 4\pi \cdot I_e \text{ and the light flux } \Phi_v = K_m \cdot 4\pi \cdot I_e$$

1.5.6 Luminance

Till now we have assumed that the radiator is dot-shaped. Now we are presuming a situation where the radiator has a surface  $a$  (Fig.21). Radiation is beamed to the solid angle  $\phi$  from this surface  $a$ . We will now define the luminance as luminous intensity per surface unit.

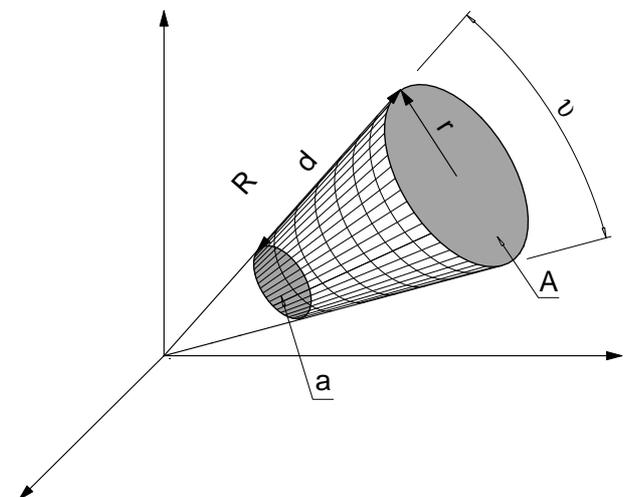


Fig.21: Surface radiator

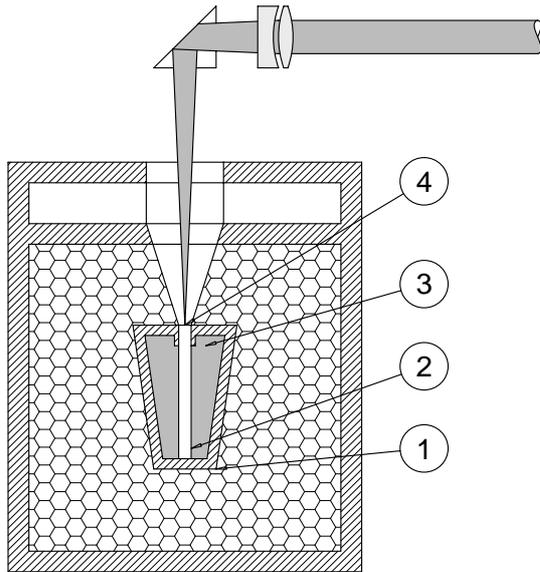
At a distance  $d$  we will measure the radiant intensity  $I_e$  under a solid angle determined by the surface  $A$ . The radiant

intensity is the same on both surfaces  $a$  and  $A$  since they belong to the same solid angle. The radiance on the radiating surface  $a$  is, however, higher than on the surface  $A$  since

$$\frac{I_e}{a} > \frac{I_e}{A} \text{ since } a < A$$

## 1.6 Practical Photometry

### 1.6.1 How the Candela is defined



**Fig.22: Diagram of the black body radiator for demonstration of the basic unit Candela .(1) crucible (2) Thorium oxide tube, (3) Platinum (4) outlet diaphragm for radiation**

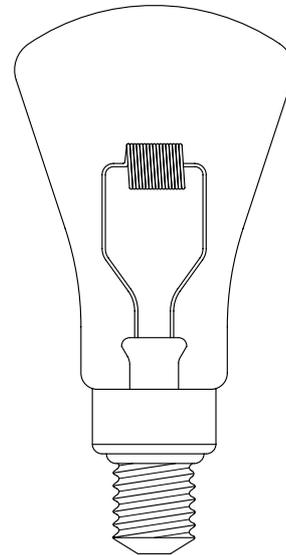
The basic unit Candela (cd) corresponds to the luminous intensity  $I$  which is produced vertically from  $1/600.000 \text{ m}^2$  of the surface of a black body radiator at the temperature of solidifying platinum ( $T = 2045 \text{ }^\circ\text{K}$ ) The principal set-up is shown in Fig.22. A crucible (1) containing platinum (3) as well as small thorium oxide tube is placed in an oven, preferably heated by hf-radiation. The small tube is the actual black body radiator and the platinum serves the purpose of defining and maintaining the temperature. A constant temperature (2.045 degrees K) is maintained during the solidifying phase due to the latent heat of platinum. The light from the black body radiator passes through the diaphragm (4) and is prepared for further use through a deviating prism as well as imaging optics. The degrees of transmission of the used optics and the solid angle taken in by the radiation, must, of course be known.

### 1.6.2 Secondary standards

As you can well imagine, it is mostly the privilege of institutes to use a radiator with platinum as a temperature refer-

## 2.1 Experimental set-up

ence since a minimum of 200-300 g of the extremely expensive platinum is required. Use of the black body radiator is also very difficult since extremely high temperatures must be controlled. For this reason, secondary standards close to the primary standard are set up through comparison. Specially made tungsten light bulbs with a high long term stability are preferable used for this purpose.



**Fig.23: Luminous intensity secondary standard W140/G**

Fig.23 shows a secondary standard. The spiral-wound filament is fixed very strongly unlike with normal light bulbs. The electrical operation data require 30 volts at a maximum current of 6 amperes. The structure of the filament can thus be constructed to produce a defined radiator with the necessary long term stability.

## 2 Experiments

In the experimental part which now follows the main objective is the measurement of the luminance. Further variables can also be measured. The measurement of the luminous intensity, however, which is much more demanding, will not be carried out. The lamp W140/G will therefore be used as a secondary standard. It is calibrated in units of luminous intensity. A pyroelectric detector with a spectral sensitivity of 1 in a range of  $0.2 - 20 \text{ } \mu\text{m}$  will be used. The detector is calibrated in radiometric units. The secondary standard is used to calibrate it to photometric units.