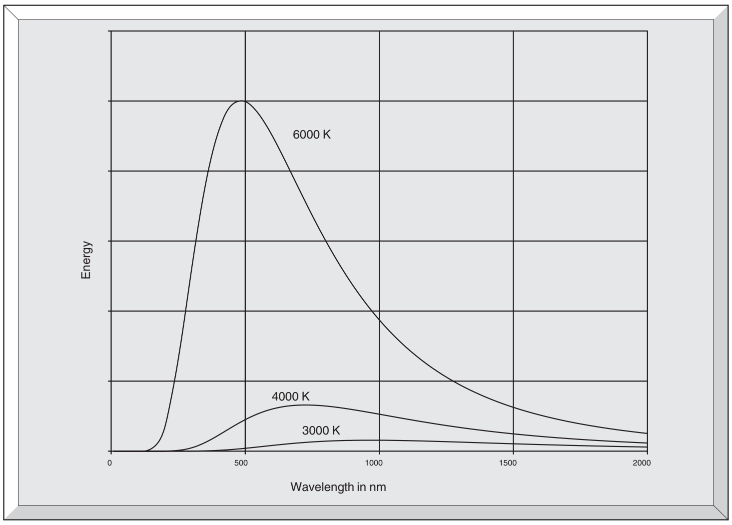
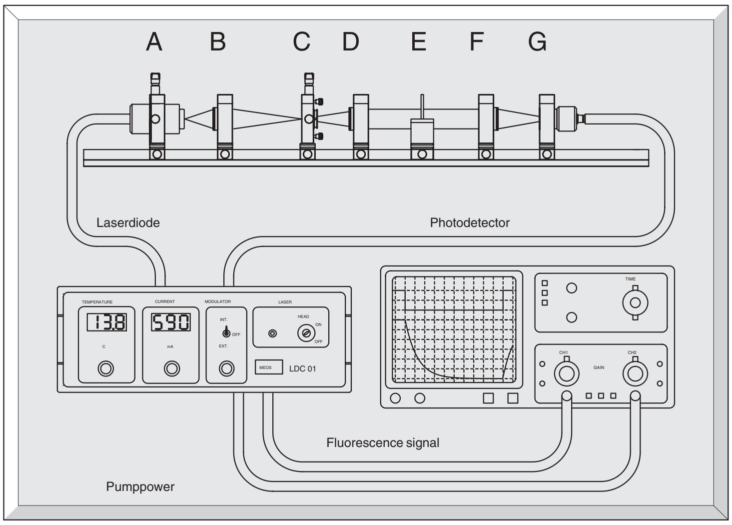
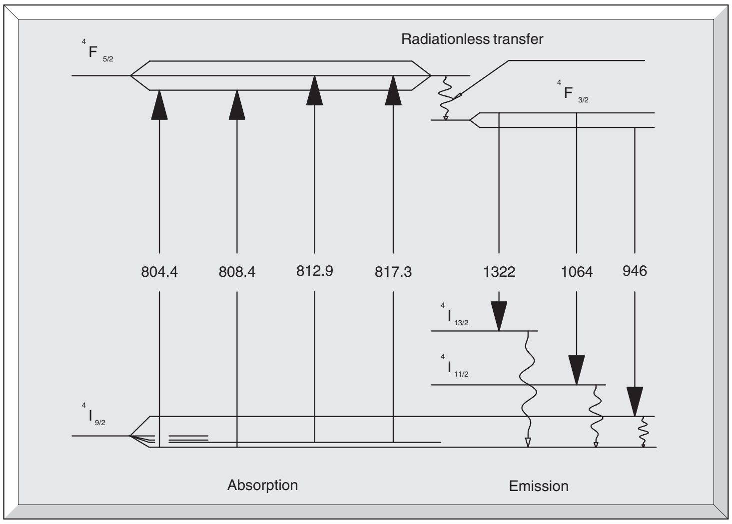


EXPERIMENT 01

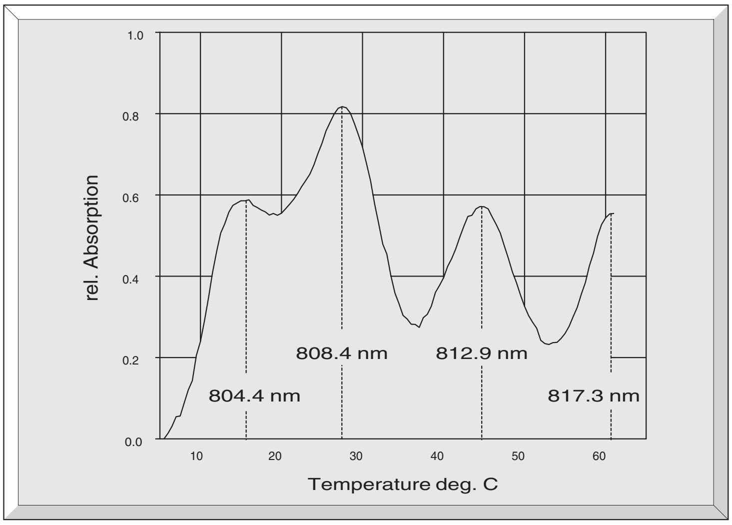
Emission and Absorption





Didactic Counsellor

Prof. Dr.-Ing. Dickmann



Fachhochschule
Münster

Fachbereich
Physikal. Technik

EXP01 Emission and Absorption

1 Fundamentals

1.1 Introduction

Light, the giver of life, has always held a great fascination for human beings. It is therefore natural that people have been trying to find out what light actually is, for a very long time. We can see it, feel its warmth on our skin but we cannot touch it.

The ancient Greek philosophers thought light was an extremely fine kind of dust, originating in a source and covering the bodies it reached. They were convinced that light was made up of particles.

As human knowledge progressed and we began to understand waves and radiation, it was proved that light did not, in fact, consist of particles but that it is an electromagnetic radiation with the same characteristics as radio waves. The only difference is in the wavelength.

We now know, that the various characteristics of light are revealed to the observer depending on how he sets up his experiment. If the experimentalist sets up a demonstration apparatus for particles, he will be able to determine the characteristics of light particles. If the apparatus is the one used to show the characteristics of wavelengths, he will see light as a wave.

The question we would like to be answered is: What is light in actual fact ? The duality of light can only be understood using modern quantum mechanics. Heisenberg showed, with his famous "Uncertainty relation", that strictly speaking, it is not possible to determine the position x and the impulse p of a particle of any given occurrence at the same time.

$$\Delta x \cdot \Delta p_x \geq \frac{1}{2}\hbar \qquad \textbf{(1.1.1)}$$

If, for example, the experimentalist chooses a set up to examine particle characteristics, he will have chosen a very small uncertainty of the impulse Δp_x. The uncertainty Δx will therefore have to be very large and no information will be given on the position of the occurrence.

Uncertainties are not given by the measuring apparatus, but are of a basic nature.

This means that light always has the particular quality the experimentalist wants to measure. We can find out about any characteristic of light as soon as we think of it. Fortunately the results are the same, whether we work with particles or wavelengths, thanks to Einstein and his famous formula:

$$E = m \cdot c^2 = \hbar \cdot \omega \qquad \textbf{(1.1.2)}$$

This equation states that the product of the mass m of a particle with the square of its speed c corresponds to its energy E. It also corresponds to the product of Planck's constant $h = \hbar \cdot 2\pi$ and its radian frequency $\omega = 2\pi \cdot \nu$, In this case ν represents the frequency of luminous radiation.

1.2 Types of luminous radiation

In the following, radiation always means electromagnetic radiation. We are particularly interested in the „optical range“. This range starts from wavelengths of 0.1 µm to approx. 1 mm. The spectral range visible to the eye plays an important part.

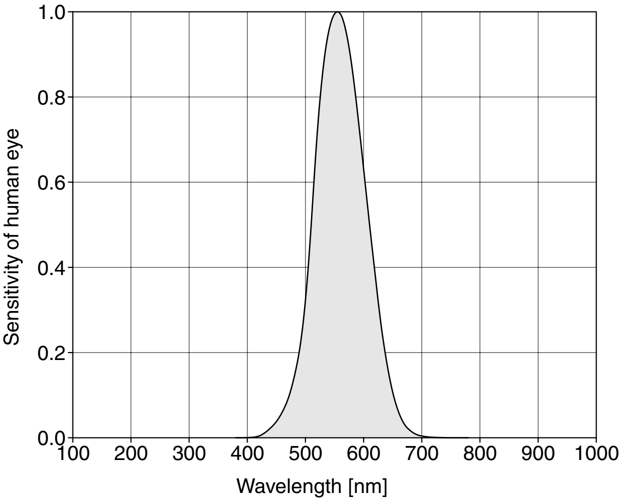


Fig. 1: Spectral sensitivity of the human eye

Whereas the quantity of radiation in the optical range is usually measured in watt seconds or watts, the visible part of the optical radiation has its own photometric quantities with the basic unit known as candela (cd). It is the sixth variable apart from the basic quantities length (m), mass (kg), time (s), electrical current (A) and temperature (T), which were laid down at the 10th International Conference for Weights and Measures in 1954.

In the following we will concentrate on the optical spectral range of electromagnetic radiation and first of all describe the basic regularities required for its deduction. In the next chapter we will examine radiation, for which the energy comes from the caloric content of a radiating body. This type of radiation is called thermal radiation and is emitted by glowing solids (metals, carbons, etc.) or gases at high pressures.

But radiation can also be produced without the energy escaping from the caloric content of a medium, e.g. by introducing an electric energy (gas discharge).

In this type of radiation production, the temperature of the radiator (except in cases of loss mechanisms) does not change. Light produced in this way is known as cold light or luminescent radiation. The type of emission depends on the electronic or vibronic states of the atoms or molecules concerned. Light sources which emit light through these light mechanisms are an important part of modern light engineering. Whereas light bulbs are thermal radiators, fluorescent lamps, gas discharge lamps, screens and lasers are sources of cold light. We will discuss the regularities of this type of radiation in Chapter 1.4. In whichever way light radiation is produced, the aim is to determine the energy E of the radiation.

1.3 Thermal Light

We know from our daily observations that a warm body emits light radiation. The higher the temperature of the body, the whiter the light. At the turn of the century, physicists were particularly preoccupied with this phenomenon. They were looking for the function giving the light energy of a radiant body as a function of the light wavelength λ and the temperature T. They presumed that the radiation energy was also influenced by the material characteristics of the body in question. Let us call these qualities P_{body}. The function

$$E\left(\lambda, T, P_{body}\right) = ? \qquad (1.3.1)$$

had to be found which corresponded to the experimental data. If we consider the numerous substances which would radiate light when heated, it would at first seem impossible to derive a universal expression for $E(\lambda, T, P_{body})$. In 1861 G. Kirchhoff made a crucial contribution by simplifying the rational examination of radiation. The experimental set up shown in Fig. 2 helps us to understand this extremely important Kirchhoff's law. The diagram shows a radiopaque covering inside which there is a ball, with a polished upper half and a blackened lower half. The inside of the covering is empty. This is to avoid effects such as convection and heat conduction. At the beginning of our experiment, both the ball and the covering should have the same temperature T. We know from experience that a black body emits more radiation than a light coloured one. We therefore expect the black half of the ball to cool off faster than the polished half. This means that there would be a difference in temperature between the black and the polished parts after a certain period of time. Therefore energy can be drawn out of both parts by attaching a thermo couple, for example, although no energy was introduced. However, this kind of "perpetuum mobile" is not possible and we will have to examine the situation more closely.

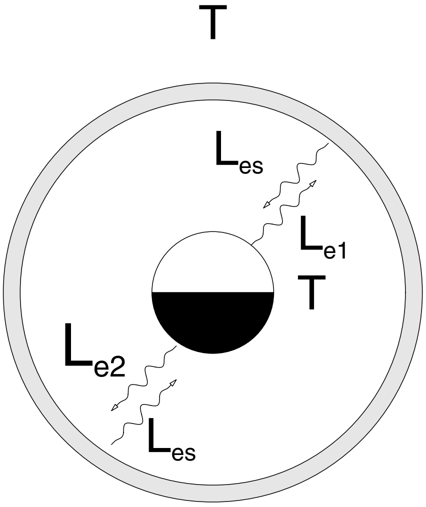


Fig. 2: Kirchhoff's Law

The ball is also exposed to the radiation field of the covering which radiates at an irradiance of L_{es}. The polished part radiates at an irradiance of L_{e1} and absorbs radiation from the covering at a degree α_1. The black ball radiates at L_{e2} and absorbs radiation from the covering at a degree α_2. The initial temperature T can only be maintained if the black part of the ball absorbs more radiation from the covering than the polished half. The following equation is applicable in this case

$$L_{e1} = \alpha_1 \cdot L_{es} \text{ and } L_{e2} = \alpha_2 \cdot L_{es} \text{ or} \qquad (1.3.2)$$

$$\frac{L_{e1}}{\alpha_1} = \frac{L_{e2}}{\alpha_2} = L_{es} \qquad (1.3.3)$$

It can be easily concluded that the same relationship applies for areas on the ball which have other values for L_e and α. This means that the relationship of the irradiance L_e of any given thermal radiator to its degree of absorption α, at the same temperature, is a constant L_{es}. This constant is physically important. In the limiting case, a black body can only have the degree of absorption $\alpha = 1$. This would then be a case of an absolute black body which absorbs all radiation.

We have presumed this in our experiment for the covering shown in Fig. 2, since the degree of absorption on the right hand side of Eq. 1.3.3 is identical with 1. It therefore applies to any temperature.

The ratio of the irradiance (L_e) over the degree of absorption (α) of any given thermal radiator is equal to the irradiance (L_{es}) of the black body at the same temperature.

$$\frac{L_e(T)}{L_e(T)_s} = \alpha(T) \qquad (1.3.4)$$

Or, to put it in another way:
The degree of emission in any given thermal radiator is the same as the degree of absorption. In the case of the absolute black body with the degree of absorption $\alpha = 1$ the degree of emission is $\varepsilon = 1$.
We have not as yet considered the wavelength of the radiation. Kirchhoff's great achievement was, that he proved that the degree of absorption is the same as the degree of emission at a certain temperature T, and also that this relationship does not depend on the wavelength λ of the radiation. Kirchhoff proved that

$$\int_0^\infty \varepsilon(\lambda, T)\, d\lambda = \int_0^\infty \alpha(\lambda, T)\, d\lambda \qquad (1.3.5)$$

or

$$\int_0^\infty \left[\varepsilon(\lambda, T) - \alpha(\lambda, T)\right] d\lambda = 0 \qquad (1.3.6)$$

The following applies for every temperature and wavelength:

$$\varepsilon(\lambda, T) = \alpha(\lambda, T) \qquad (1.3.7)$$

The importance of this is stated in the following:

The radiation of any given thermal radiator is dependent on its absorption capacity, but not on any of its other specific qualities.

Thanks to Kirchhoff's work the expression we were looking for (1.3.1) can now be simplified to

$$E(\lambda, T) = ? \qquad \textbf{(1.3.8)}$$

Kirchhoff has also given details in his work done around 1861, on how to construct an ideal black body radiator, which absorbs radiation at all wavelengths and therefore has the highest possible degree of emission of a thermal radiator. He says:

"If a space is filled with bodies of the same temperature and no radiation can penetrate these bodies, the quality and intensity of the radiation cluster within the space is such that it seems to originate in a perfect black body of the same temperature and therefore does not depend on the constitution and form of the bodies, but only on the temperature."

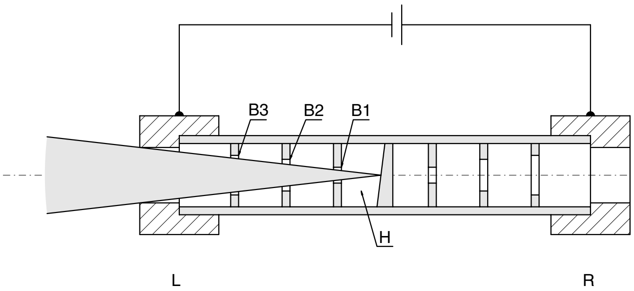


Fig. 3: Black body radiator

Fig. 3 shows the set up of a black body radiator. A tube made of tungsten, for instance, has a cavity H, from which radiation is passed out through the diaphragms B1 to B3. The tube itself is fixed into the copper rings R and L through which the electric current for heating the chamber is introduced simultaneously. The temperature of the cavity is determined by suitable means on the right hand side R. Thermo couples can be used according to the temperature or the radiation law can be applied, which we will deduce shortly. As simple as the experimental set up of Fig. 3 may seem, the derivation of the formula for spectral energy distribution E(λ,T) was the starting point for quantum mechanics, a new kind of physics, which caused great excitement at the beginning of this century. At that time, people had tried to find out the spectral distribution of light energy emitted by a black body theoretically. Lord Rayleigh (Philos.Mag. Vol.49 (1900)P.539) and J.H. Jeans (Philos.Mag.Vol.10 (1905) P.91) used a simple model for the absorption and emission mechanism as a basis, which stemmed from the laws of statistics of the gas theory and other thermodynamic disciplines applicable till then. Both authors calculated that the intensity of the radiation emitted by the black body radiator was proportional to the square of the frequency of the radiated light. According to the most elementary experiences, however, the intensity at high frequencies (UV light) is close to zero. All attempts to eliminate this contradiction using classical physics (there was no other kind at the time) failed. It was an absolute catastrophe. Try and imagine what physicists must have felt at that time. After thinking that physics was a complete science and nature could be interpreted without any problems, this was a shock that caused many doubts.

1.3.1 Radiation law u(λ,T)

Max Planck (Verh. dtsch. physik. Ges.(14.Dec.1900) and Ann. Physik(4)Vol.4(1901)P.553-564) found a solution to this UV catastrophe (still called so today) with a totally new hypothesis according to which the absorption and emission processes should be carried out in an elementary way. The strange relationship, where the energy in such an elementary process is proportional to the light frequency is still applicable.

$$E = h\,\nu \qquad \textbf{(1.3.9)}$$

The proportional factor h is known as Planck's constant or the quantum of action. Planck was able to formulate a radiation law with the help of this hypothesis which has proved successful till today. Planck determined this constant, h = 6.57 10^{-34} Ws^2 through comparative measurements. Today's value is 6.626196 10^{-34} W s^2. Planck laid the foundations for quantum physics with his hypothesis. As we know from history, it took a long time for classical physicists to admit that classical physics is only marginal to quantum physics.

Let us now return to the derivation of the function E(λ,T), which was mainly influenced by Rayleigh and Jeans but finally formulated by Planck. We will imagine the full radiator to be a cube with the edge length L

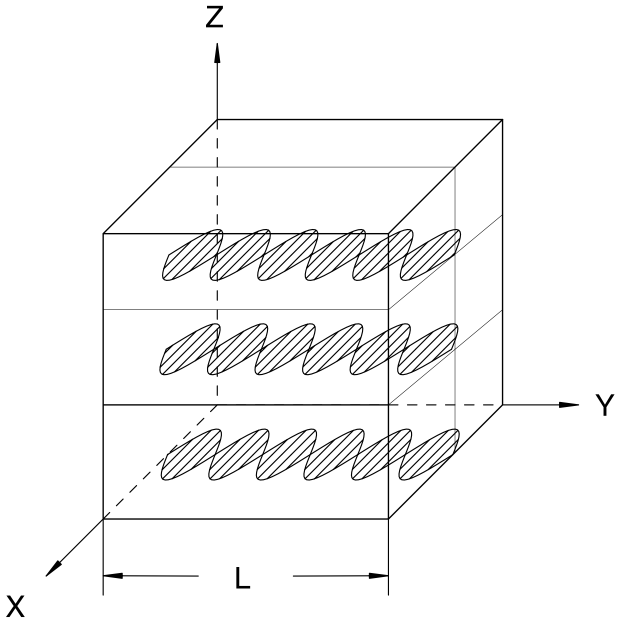


Fig. 4: Cavity resonator with standing light waves

We found out in Chapter 1.3 (Fig. 2) that the inner walls of the black body radiate and absorb. There is obviously a radiation field within the box which consists of standing waves since there is a reverse radiation to the wall and from the wall. We must first try and answer the question, how many standing light waves of a particular wavelength λ are present in a box with edge length L . We will carry out the same procedure for every possible wavelength. The possible number of standing waves in each case will give us the answer to the question, how much energy does the particular number of standing waves have . This type of box has been shown in Fig. 4.

Only standing waves that can fit into the box are formed, i.e. the distance between the nodes must be integral to the length L. Since the distance between the nodes is just about half that of the wavelengths, the condition

$$L = n\frac{\lambda}{2} \qquad (1.3.10)$$

must be fulfilled. In this case n is a whole number and λ is the wavelength of light radiation. In Fig. 4 only the waves formed in the direction of the Y-axis are shown. The same is applicable to the Z or X axis. We must also take the possibility of a three dimensional standing wave, as shown in Fig. 5, into consideration.

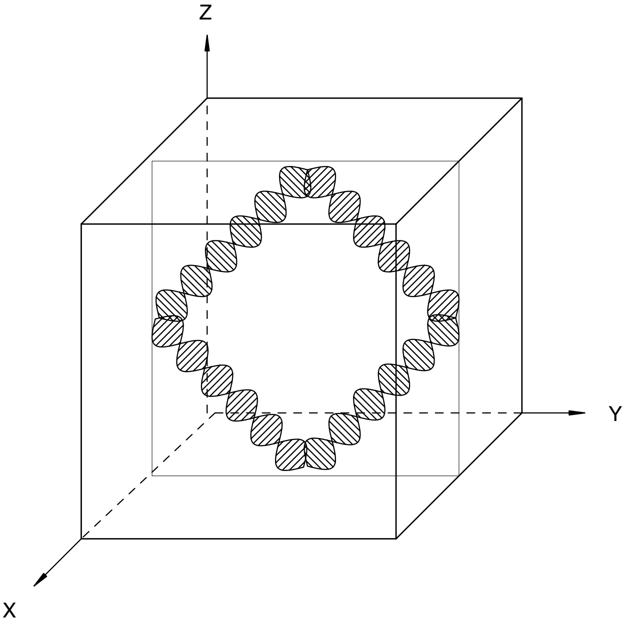


Fig. 5: Spatial standing wave propagating in the ZY-plane

At this point, if not before, the wave vector must be introduced:

$$\vec{K} = \vec{e} \cdot \frac{2 \cdot \pi}{\lambda} \qquad (1.3.11)$$

The vector $\vec{K}$ points in the direction of propagation of the light wave. $\vec{e} =$ (e_x,e_y,e_z) is its directional vector where $|\vec{e}| = 1$.The amount of $\vec{K}$ is also called the wave number:

$$|\vec{K}| = \frac{2 \cdot \pi}{\lambda} \qquad (1.3.12)$$

The dimension of the wave number is m^{-1} and for practical reasons spectroscopists would prefer this to be in cm^{-1}. The propagation of light can be shown and described in an easier way because of the introduction of the wave vector. This is clarified in Fig. 6, which shows the situation of Fig. 5 using wave vectors.

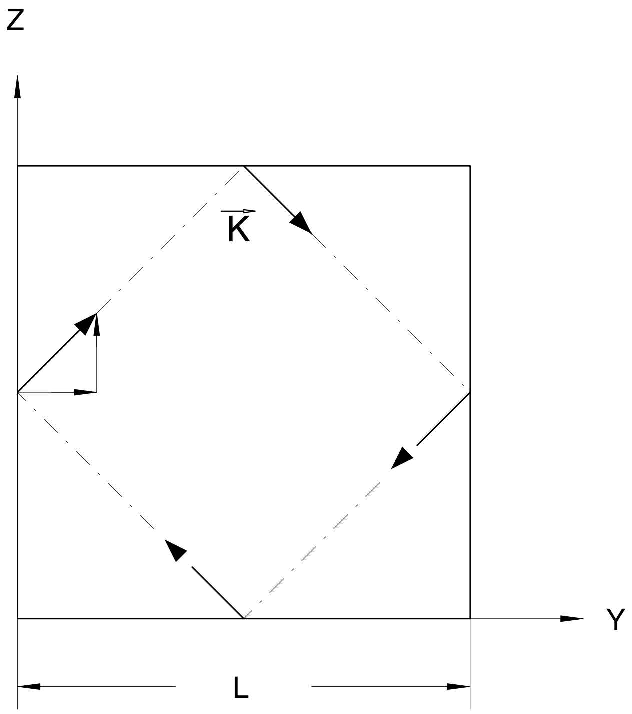


Fig. 6: Like Fig. 5 but representation with wave vectors

According to Eq. 1.3.10 the condition L = nλ/2 must be fulfilled if standing waves are to be formed. We can see in Fig. 6 that the wave vector is represented by its orthogonal components. Therefore the following condition is applicable for the formation of standing waves in any given direction in space:

$$\vec{K} = (n_1, n_2, n_3) \cdot \frac{\pi}{L} \qquad (1.3.13)$$

and

$$|\vec{K}| = K = \frac{\pi}{L} \cdot \sqrt{(n_1^2 + n_2^2 + n_3^2)} \qquad (1.3.14)$$

If we now use the relationship between the wavelength λ, the frequency ν and the speed c of light waves

$$\nu = \frac{c}{\lambda} \text{ or with } \omega = 2\pi \cdot \nu \qquad (1.3.15)$$

the frequencies of the standing waves can also be determined by:

$$\omega = \frac{\pi \cdot c}{L} \cdot \sqrt{(n_1^2 + n_2^2 + n_3^2)} \qquad (1.3.16)$$

What has been achieved till now ? We know that standing light waves - or to put it differently - a stationary radiation field, can be formed in a cubic box (resonant chamber). This involves discrete, integral values n, which show how many oscillation nodes are formed at a given length L of the cavity and at a particular wavelength or frequency. Every n or n_i fulfilling Eq.1.3.16 marks a particular state of the radiation field. This state defined by n is called the oscillation mode or simply“ mode“. The answer to the question, how many such modes exist in a particular frequency interval provides the spectral energy distribution of the emission of a black body radiator. But first we have to find out the number of possible modes. This is done simply by counting them. We will use a trick for this purpose that is often used in statistical physics and which is shown in Fig. 7.

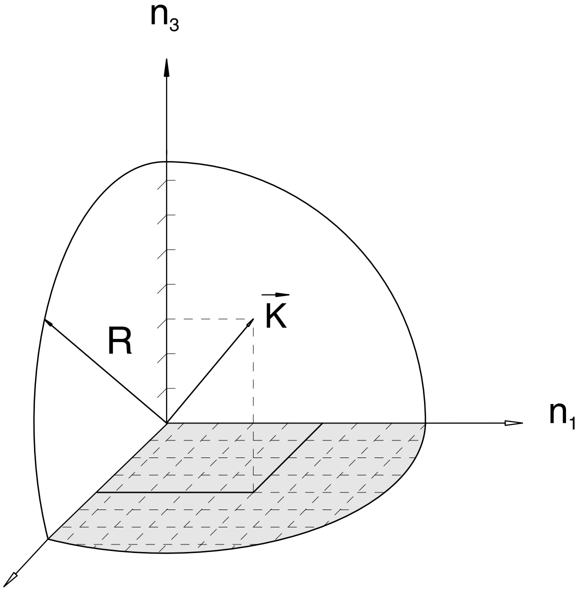


Fig. 7: Representation within the wave vector coordinate system

If we remember the equation with a sphere with radius R

$$R = \sqrt{x^2 + y^2 + z^2} \qquad (1.3.17)$$

and write Eq.1.3.14 in the form

$$\frac{K \cdot L}{\pi} = \sqrt{n_1^2 + n_2^2 + n_3^2} \qquad (1.3.18)$$

then compare it to Eq.1.3.17 the result will be an analogous equation.

We can take Eq.1.3.18 as an equation for a sphere. The radius of this sphere is K L/π or 2L/λ and n_1 n_2 n_3 represent the x y z coordinates. Because the n's are integral and positive they can produce only an eighth of the full sphere, formed from a three dimensional lattice with the lattice constant 1.

Therefore only those wave vectors $\vec{K}$ whose components coincide with the n values are permissible, or to put it differently, every intersecting point of the lattice produces a valid solution for a wave vector $\vec{K}$ of a standing wave. The answer to the question posed at the beginning, regarding the number of modes required for a given length L of the resonant cavity can, in fact, be found after simply counting the intersecting points of the lattice. This can also be done analytically, using the formula for the volume of a sphere.

$$V_{sphere} = \frac{4}{3} \cdot \pi \cdot R^3 \qquad (1.3.19)$$

For an eighth of a sphere with a "radius" for an upper frequency limit at $\omega_{\max}$.

$$N(\omega_{\mathrm{max)}}) = \frac{1}{8} \cdot \frac{4}{3} \pi \cdot \left(\frac{L \cdot \omega}{\pi \cdot c} \right)^3 \qquad (1.3.20)$$

Since vertically polarised light is not mutually influencing the number N of modes increases by a factor of 2 and we obtain the following solution:

$$N(\omega_{\mathrm{max)}}) = \frac{\pi}{3} \cdot \left(\frac{L \cdot \omega}{\pi \cdot c} \right)^3 \qquad (1.3.21)$$

If we divide Eq.1.3.21 by the volume L^3 of the resonant cavity, we obtain the mode density $n(\omega_{\max})$

$$n(\omega_{\max}) = \frac{\pi}{3} \cdot \left(\frac{\omega}{\pi \cdot c} \right)^3 = \frac{8\pi}{3} \left(\frac{\nu}{c} \right)^3 \qquad (1.3.22)$$

By differentiating after dν, we get the mode density in a frequency interval dν

$$dn(\nu_{\max}) = \frac{8\pi}{c^3} \cdot \nu^2 d\nu \qquad (1.3.23)$$

Up to this point, calculations were made according to Rayleigh. Rayleigh then followed the laws of the kinetic gas theory to obtain the spectral energy. When using this law the energy $E_m = 1/2kT$ is attributed to every degree of freedom of a system in equilibrium. In this case k is the Boltzmann constant and T the absolute temperature. The average kinetic energy in harmonic oscillations is equal to the average potential energy. Therefore the energy per degree of freedom for harmonic oscillation processes is kT. Rayleigh's result, the so called UV catastrophe, was as follows:

$$u(\nu, T) = dn(\nu_{\max}) \cdot k \cdot T$$

$$u(\nu, T) = \frac{8\pi}{c^3} \nu^2 \cdot k \cdot T \cdot d\nu \qquad (1.3.24)$$

Of course, Rayleigh knew that his results contradicted the experiments, but they corresponded perfectly to the laws of physics known up to that time. The starting point for these laws is the Boltzmann distribution.

1.3.2 The Stefan-Boltzmann Laws

Ludwig Boltzmann lived from 1844-1906 and is the founder of the atomic theory and electrodynamics. In 1879 J. Stefan deduced, from measurements taken by French physicists, that the density of energy for all frequencies of a resonant cavity is the same and equal to a constant multiplied by the temperature to the power four.

$$E(T) = k\,T^4 \qquad (1.3.25)$$

Stefan presumed that this law applied to all radiators. In 1884 Boltzmann limited it to black bodies and finally derived and formulated it. It has been confirmed by measurements and its fundamental importance lies in the proof that a volume filled with radiation can be treated as if there were a gas in the volume.

Boltzmann got his results by using statistical methods. Just as a probability is calculated by cubing a certain number, Boltzmann solved the problem of how many particles of a gas dissolve out of the total quantity N at a temperature T within an energy interval dE. His result was the important Maxwell-Boltzmann distribution:

$$\frac{dn}{N} = \frac{\sqrt{8/\pi}}{\sqrt[3]{k^2 T^2}} \sqrt{E} \cdot e^{-\frac{E}{kT}} \cdot dE \qquad (1.3.26)$$

Boltzmann obtained as average energy for all three degrees of freedom of a gas molecule using Eq.1.3.26.

$$E_m = 3/2\, k\, T \qquad \textbf{(1.3.27)}$$

and thus found the T^4 law (The Stefan-Boltzmann Law) which was also correct for the resonant cavity. Lord Rayleigh used these findings, which were proved to be right. But this still led to the UV catastrophe, which Planck solved with his new hypothesis and which showed that Rayleigh was right, but only under certain conditions.

1.3.3 Planck's Radiation Law

Max Planck carried out the classical calculations once more, but this time he intuitively understood that only energies that fulfilled the condition

$$E = n\, h\, \nu \qquad \textbf{(1.3.28)}$$

were permissible.

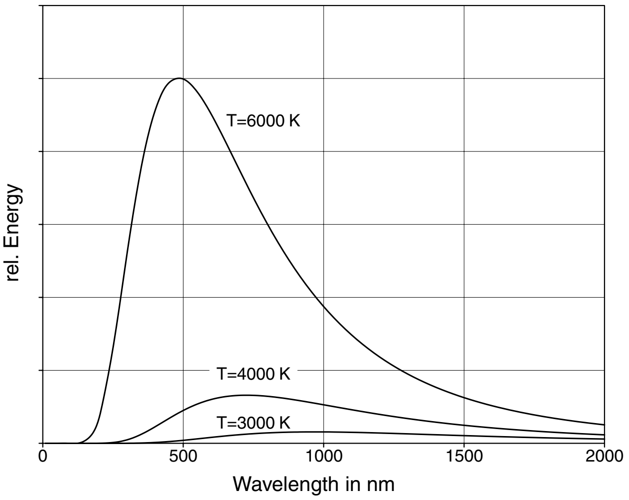


Fig. 8: Planck’s law of radiation for different temperatures of the black body radiator

Without being aware of the future consequences, i.e. the development of a new kind of physics called quantum physics, Planck maintained that the radiation energy was not distributed continuously, as physicists had correctly assumed till that point, but in an integral quanta n. However, a constant h was necessary for the required dimension of energy on the right hand side of Eq.1.3.28. Boldly assuming this, he calculated that the average energy E_m was no more kT as Rayleigh had assumed based on Boltzmann's work, but

$$E_m = \frac{h \cdot \nu}{e^{-\frac{h \cdot \nu}{k \cdot T}} - 1} \qquad \textbf{(1.3.29)}$$

If this is multiplied with (1.3.23) the energy density u(ν,T), analogous to the way Rayleigh proceeded, would now result in

$$u(\nu, T) = \frac{8\pi}{c^3} \cdot \frac{h \cdot \nu^3}{e^{-\frac{h \cdot \nu}{k \cdot T}} - 1} \cdot d\nu \qquad \textbf{(1.3.30)}$$

With ν = c/λ Eq.1.1.30 is a function of the wavelength

$$\boxed{u(\lambda, T) = \frac{8\pi \cdot c}{\lambda^5} \cdot \frac{h}{e^{-\frac{h \cdot c}{\lambda k \cdot T}} - 1} \cdot d\lambda \qquad \textbf{(1.3.31)}}$$

This is the famous Planck Radiation Formula, and we have now completed the derivation of the energy of a full thermal radiator as a function of its temperature and the distribution of energy on wavelengths emitted by the radiator. In Fig. 8 this function has been calculated and graphically represented for three temperatures. If we analyse the dimensions of u(λ,T) we will get Ws/m^3 or energy per unit volume of the black radiator

1.4 Cold Light

We have learned about one kind of light production and have also described the radiation of bodies at a temperature T.

At the same time as these examinations and calculations were being carried out, spectroscopists were working on the meaning of light emission and the absorption of atoms and molecules which, contrary to the continuous radiation of the thermal radiator showed a structure that was characteristic for certain atoms and molecules.

Apart from its function as a source of light, physicists were primarily interested in the meaning of spectra, since they could find out more on the structure of atoms and molecules in this way.

After the research into the physics of atoms also came to standstill, Niels Bohr took up Max Planck's hypothesis. He expanded his existing theory on the atom model and said that the light emitted or absorbed by atoms could only have an energy of E_2-E_1 = hν (Fig. 9).

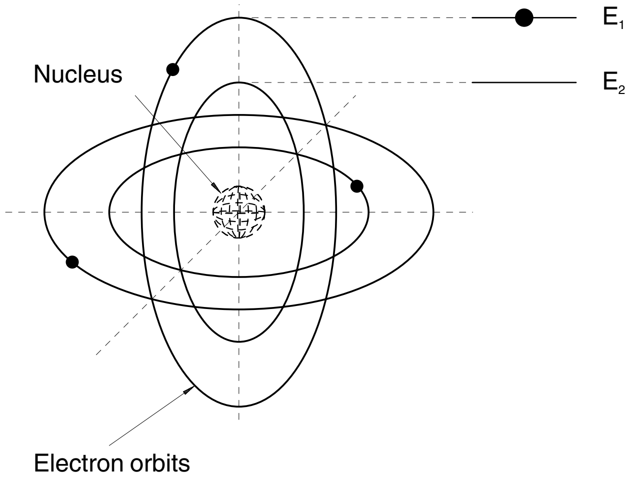


Fig. 9: Bohr's Atom

The work and measurements had proved that discrete energy must be anticipated for both resonant cavities and appearances of atomic emissions. Einstein began looking for a single description for both these sources of light. He was able to solve this problem in 1917 when he derived Planck's hypothesis once more in his own way. He thought of a way of combining both light sources. Instead of a sphere, like the one we used in Fig. 2, he put an ensemble of "Bohr" atoms in the resonant cavity at a temperature T.
In thermal equilibrium E(ν,T) will be an energy distribution which must be marked by the qualities of the atoms as a "cold light source".

The new distribution E(ν,T) must be found. The first step is to examine the atom ensemble, which we presume has only two energy levels, as shown in Fig. 9.

Since the atoms are in a radiation field, they can take up or absorb energy. The absorption is connected to an emission. If we call the number of electrons in state 1, n_1, the temporal change in this quantity will be

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u(\nu) \qquad \textbf{(1.4.1)}$$

In this case u(ν) is the density of energy at the frequency at which the transition from state 1 to state 2 is resonant, i.e. it is the frequency at which E_2-E_1=hν is fulfilled.
This frequency is called the resonant frequency. It is evident that the temporal change from dn_1/dt is dependent on the number n_1 itself, on one hand, and on the density of energy of the radiation with the frequency ν, on the other.

A constant B_{12} is necessary for a correct equation in terms of dimension. The minus sign is required because the number of electrons in state 1 decreases through the absorption .
The same observation is carried out for state 2. We will call the number of electrons in this state n_2. The electrons return to state 1 from state 2 whilst emitting radiation. The transition from 2 to 1 is released (induced) by the existing radiation field of the resonator and takes also place coincidentally (spontaneously). So, two types of emission are responsible for depopulating state 2, the induced and the spontaneous emission. The temporal change in the number n_2 is

$$\frac{dn_2}{dt} = -B_{21} \cdot n_2 \cdot u(\nu) - A_{21} \cdot n_2 \qquad \textbf{(1.4.2)}$$

Nothing has been left out of the last term since the spontaneous emission does not depend on the surrounding radiation field and is of a statistical nature. It takes place even when there is no radiation field. Until the principles of quantum mechanics were defined by Heisenberg and Schroedinger, it was accepted that spontaneous emission was similar to radioactive decay, in that it could not be influenced from the outside. Quantum electrodynamics has shown that a spontaneous emission is an emission induced by zero point energy. So as not to take this too far at this point, the following must be noted with reference to zero point energy.

In the cavity there is an average field energy of at least E_0=1/2 hν. The spontaneous emission is triggered off by this energy.

Let us go back to our resonant cavity-two level atom system. In stationary equilibrium, the same number of electrons must go from state 1 to 2 (with a photon being absorbed from the radiation field) and vice-versa (emission of a photon into the radiation field)

$$\frac{dn_1}{dt} = \frac{dn_2}{dt} \qquad \textbf{(1.4.3)}$$

or

$$B_{12} \cdot n_1 \cdot u(\nu) = B_{21} \cdot n_2 \cdot u(\nu) + A_{21} \cdot n_2 \qquad \textbf{(1.4.4)}$$

The Boltzmann distribution is also valid in the thermal equilibrium for the population numbers of level 1 and level 2

$$n_2 = n_1 \cdot e^{-\frac{E_2 - E_1}{k \cdot T}} \text{ or } n_2 = n_1 \cdot e^{-\frac{h \cdot \nu}{k \cdot T}} \qquad \textbf{(1.4.5)}$$

By substituting (1.4.5) in (1.4.4) we get

$$u(\nu, T) = \frac{A_{21}}{B_{12}} \cdot \frac{1}{e^{-\frac{h \cdot \nu}{k \cdot T}} - B_{21} / B_{12}} \qquad \textbf{(1.4.6)}$$

Since Planck's law must be valid also in equilibrium we get by comparison of (1.3.33) with (1.4.6) the significant Einstein coefficients:

$$B_{12} = B_{21} \text{ und } \frac{A_{21}}{B_{12}} = 8\pi \cdot \frac{h \cdot \nu^3}{c^3} \qquad \textbf{(1.4.7)}$$

1.4.1 Natural line width

Let's look on Eq. **(1.4.1)**:

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u(\nu)$$

B_{12} can be considered as the probability for a transition from level 1 to level 2 by absorption.
This is also analogous to the coefficient B_{21}, which however indicates the probability of the reverse process, i.e. the emission.
The coefficient for the spontaneous emission A_{21} gives us another interesting piece of information on the system, which is easy to find.
Let us take, for example, the process of the spontaneous emission by itself.

$$\frac{dn_2}{dt} = -A_{21} \cdot n_2 \qquad \textbf{(1.4.8)}$$

This differential equation can be solved using the additional equation

$$n_2(t) = C \cdot e^{-\alpha \cdot t} \qquad \textbf{(1.4.9)}$$

$\alpha = A_{21}$ can be found by comparing the two and the solution will then be

$$n_2(t) = n_2(t=0) \cdot e^{-A_{21} \cdot t} \qquad \textbf{(1.4.10)}$$

Fig. 10 shows this function graphically. This curve and A_{21} can be determined experimentally. The time t which $n_2(t)$ took to gain the value n_2 (t=0) e^{-1} must be deduced. The result will then be $t = 1/A_{21}$. Obviously the reciprocal value of the Einstein coefficient A_{21} represents a suitable definition for the "life time τ of a state".

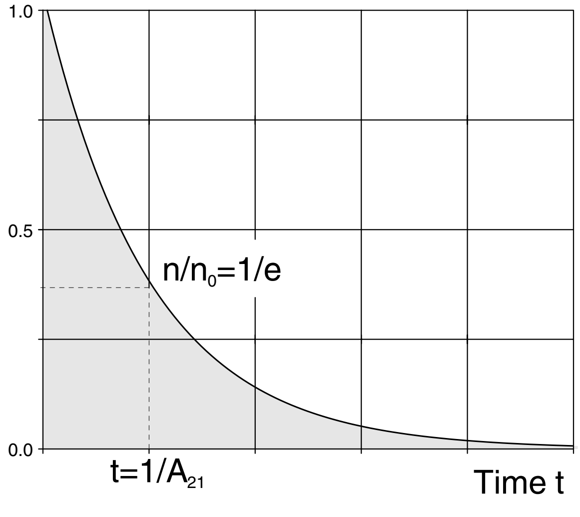


Fig. 10: Population decay curve of a state

More information can be obtained from the decay curve. Photons or a radiation field are produced because of the transition from state 2 to 1. However, the intensity of the radiation decreases exponentially with the time (Fig. 11). In view of the preceding findings, the frequency of the radiation should be fixed to E_2-E_1 = h ν_0.

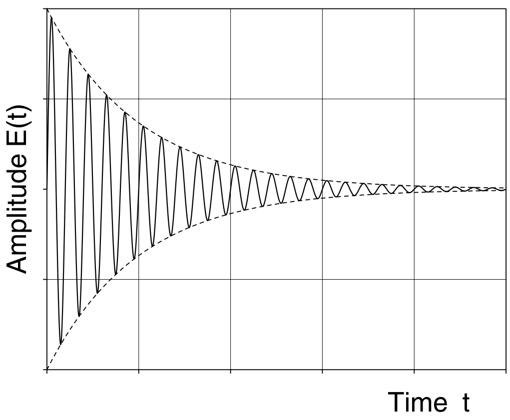


Fig. 11: Spontaneous emission as a damped oscillation

A power spectrum of the spontaneous emission is obtained using a Fourier analysis for non-periodic processes, which has the main frequency ν_0 apart from other frequency parts. The result of this kind of Fourier transformation is illustrated in Fig. 12.

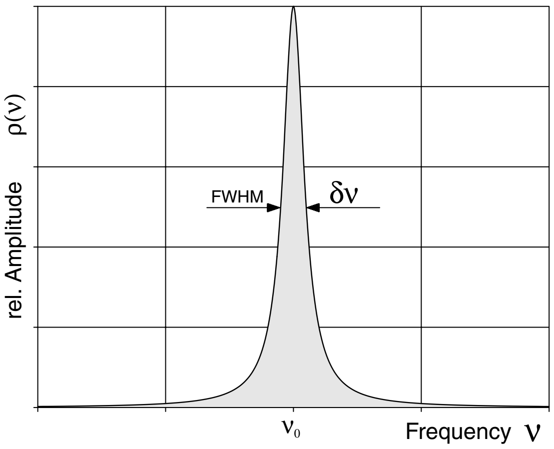


Fig. 12: The Fourier transformation of a damped oscillation as observed for spontaneous emission. It consists of the transition frequency ν_0 and a complete spectrum described by a Lorentz function.

The Fourier transformation of the damped oscillation gives the following result:

$$\rho(\nu) = \frac{1}{4\pi^2 \cdot (\nu - \nu_{21})^2 + (1/2 \cdot \tau_s)^2} \qquad \textbf{(1.4.11)}$$

This type of curve represents a Lorentz curve. ν_{21} (also called ν_0) is the resonant frequency and $\tau_s = \frac{1}{A_{21}}$ the average life time of state 2.
The FWHM (Full Width at Half Maximum) of the curve as shown in Fig. 12 is calculated by inserting the value of $\rho(\nu) = 1/2$. The result is:

$$\delta(\nu)_{nat} = \frac{1}{2\pi} \cdot A_{21} \qquad \textbf{(1.4.12)}$$

which is the natural line width of a transition, defined by the Einstein coefficient A_{21} which has a particular value for every transition. The results obtained can also be interpreted as if the state 2 did not have any clearly defined energy, but a broadening with half-width $\Delta E = 2\pi h A_{21..}$ This means that the state is somewhat blurred. Quantum mechanics has shown this effect to be extremely important. It is known as the Heisenberg uncertainty principle, after the person who found it. In the case of normal optical transitions the value of τ_s lies between 10^{-8} to 10^{-9} seconds. This life time, determined by spontaneous transitions alone, is crucial for the so called **natural width** of a spectral line. To clarify the ways in which we term things, we must emphasise briefly at this point, that there is a difference between the width of a state and the width of a line, as well as between the terms state and line. There are always states for atoms and it is never stated whether the state is occupied or empty. A line is only formed if an emission is caused by the transition from, for example, state 2 to 1. The line is a word commonly used by spectroscopists. They use their spectroscopes to produce photographic plates, for example, on which fluorescent light is shown according to its wavelengths. The use of slits in the optical beam path makes it easier to evaluate the spectra. A line spectrum of this kind is shown in Fig. 13.

Fig. 13: Recording the emission of a light source with corresponding energy levels results in a line spectrum

Apart from the emission wavelengths, the spectrum in Fig. 13 shows the line widths. It must be observed, in this case, that the measuring apparatus makes the line widths seem wider than they actually are. Naturally, it was the aim of spectroscopists to create instruments which could give the closest reading of the actual line widths.

1.4.2 Line broadening

According to the preceding account, transitions are only possible if the energy of the absorbed or emitted photon lies within the natural width of the transition.

Actually, nature is not so critical and we can observe that even photons take part in processes which deviate slightly in energy. This is because the energy levels are broadened in addition to the natural line width, due to various mechanisms. The natural line width is actually very small compared to other broadening effects. There is a broadening dE which depends on the temperature of the atoms and how they are influenced by the interaction with their environment. This leads to the acceptance of photons within this area. The width of the transition is indicated by its FWHM in each case. All of this also applies to the emission.

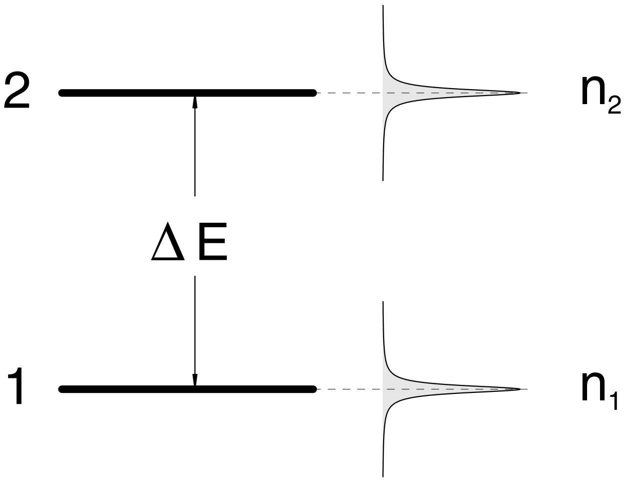


Fig. 14:" Uncertainty" of energy levels because of the natural line width

1.4.3 Homogeneous line broadening

A line is homogeneously broadened when all the atoms or molecules have the same characteristics and all of them interact with their environment in the same way. The natural broadening is a homogeneous broadening, since it is the same for all atoms and molecules in an ensemble. Homogeneous broadening can be found in solids with regular crystal structure in which the atoms considered are in equivalent lattice sites. The interactions with the crystal lattice lead to a broadening of the states that is far beyond the natural width, but which is homogeneous when the lattice sites are symmetrical and of equal value. Gases are known for their inhomogeneous broadening and this will be discussed in the next section. In this case the absorption and emission lines are no more homogeneously but inhomogeneously broadened.

1.4.4 Inhomogeneous line broadening

Materials in which atoms and molecules have different qualities do not behave in the same way as solids. This is particularly obvious in gases where atoms or molecules move more or less freely at varying speeds v. The number of particles n(v) in a total quantity of N with a mass m within a speed interval of v to v+dv is described by the Maxwell-Boltzmann distribution

$$\frac{n(\mathrm{v})}{\mathrm{N}} = \frac{4\mathrm{v}^2}{\sqrt{\pi}} \cdot \frac{1}{\sqrt[3]{\left(2kT/m\right)^2}} \cdot e^{-\frac{m\mathrm{v}^2}{kT}} d\mathrm{v} \qquad (1.4.13)$$

The above equation applies to all directions in space. To make things simple, we will take into consideration only one direction, e.g. the direction x :

$$\frac{n(\mathrm{v_x})}{N} = \sqrt{\frac{2 \cdot k \cdot T}{m}} \cdot e^{-\frac{m \cdot \mathrm{v_x^2}}{\mathrm{k \cdot T}}} \cdot d\mathrm{v}_x \qquad (1.4.14)$$

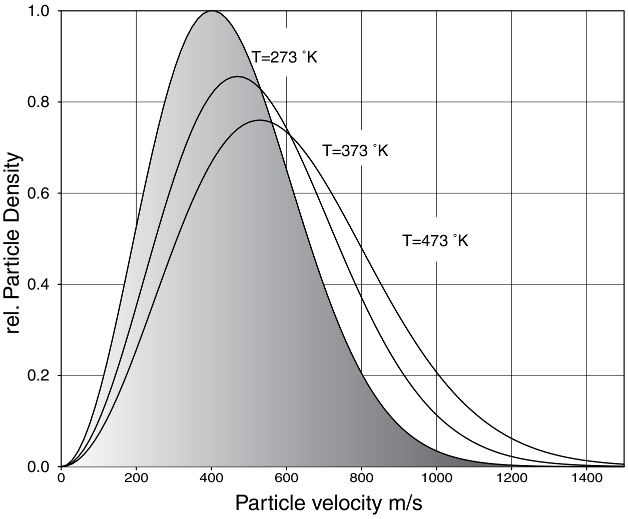


Fig. 15: Distribution of N_2 molecules according to their speed

1.4.4.1 Doppler broadening

The passive observer will now see a shift in the absorption or emission frequency due to the speed of the particles. The value is

$$\nu = \nu_0 \, (1 \pm v / c) \qquad (v << c),$$

ν_0 is the absorption or emission frequency of the resting particle and c is the speed of light. This phenomenon has been named the Doppler effect after its discoverer (Ch. Doppler, Abh. d. K. Boehmischen Ges. d. Wiss.(5), Vol.II (1842) P.465).

If Doppler's equation is inserted in the equation for the distribution of speed $n(v_x)$, the line broadening produced by the movement of particles is obtained. Since the intensity is proportional to the number of emitting particles the distribution of intensity will be

$$I(\nu) = I(\nu_0) \cdot e^{-\frac{c^2}{v_w^2} \cdot \frac{(\nu-\nu_0)^2}{\nu_0^2}} \qquad (1.4.15)$$

v_W is the most probable speed in this case

$$v_w = \sqrt{\frac{2 \cdot k \cdot T}{m}} \qquad (1.4.16)$$

The calculation for the FWHM is done in the same way as with the natural line width and the result is:

$$\Delta\nu_{doppler} = \sqrt{4 \cdot \ln 2} \cdot \frac{\nu_0}{c} \cdot \sqrt{\frac{2 \cdot k \cdot T}{m}} \qquad (1.4.17)$$

This result shows us that the line broadening is increasing due to the Doppler effect when there are higher frequencies ν_0 or smaller wavelengths (IR) and higher temperatures. The line broadening is smaller when the mass m of the particles increases.

The line profile corresponds to a Gaussian distribution. Fig. 16 shows this kind of profile.

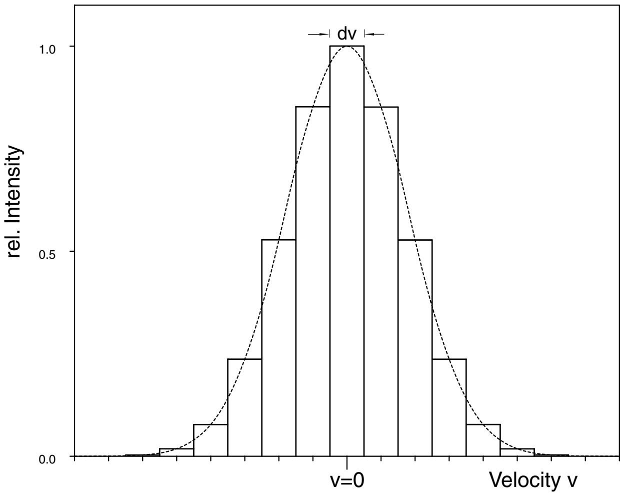


Fig. 16: Inhomogeneous line profile with speed intervals dv

The histogram is close to the smooth curve when the speed intervals dv are small. You may wonder what the purpose of these speed intervals dv or frequency intervals dν is. Strictly speaking this has formal reasons. All the results given till now are based on statistical statements. Reducing an interval to zero would be like talking about taking out a particle with an exact speed or frequency from an ensemble. It is almost impossible for this to succeed. So, the concept of intervals was introduced to describe the intervals on the basis of definite probabilities. If this is achieved the intervals can be made smaller and we can work with „smooth curves“.

Thanks to Einstein's work we now have a common description for thermal and cold radiators. Both these types of "lamps" are indispensable in modern technology. The incandescent lamp, which provides light to us daily is a thermal radiator. Technically, cold radiators such as fluorescent or arc lamps are some of the most important light sources with which high luminous powers are produced to light up large areas. We must not fail to mention the laser, another important cold radiator, which, however generally does not count as a light source because of its special characteristics. In actual fact, a cold radiator emits a line spectrum with a comparatively narrow band width, consisting of a number of lines. Radiators with an emission suited to the sensitivity of the human eye (Fig. 1) are wanted for lighting purposes. This is why the pressure in this type of lamp is increased. The lines are then broadened and the desired emission spectrum is adjusted. The effect causing line broadening due to increased pressure is known as pressure broadening and is explained in the next chapter 1.4.4.2.

1.4.4.2 Pressure broadening

Let us first observe the situation where the atoms are wide apart from each other. This is the case at pressure P_1. As shown in chapter 1.4.1 the emission spectrum consists of narrow lines superimposed by the unavoidable Doppler broadening.

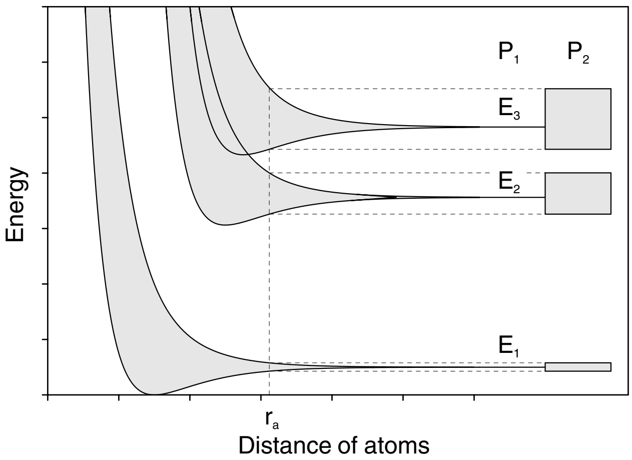


Fig. 17: Line broadening by atom approach caused by increasing the pressure from P_1 to P_2

If we increase the pressure to P_2, the number of atoms per unit volume will increase. Now the atoms are packed more densely together. There energy states will change at this point, due to the interaction of their electrons. The average distance r_a of the atoms depends on pressure and temperature of the gas used. Unlike regular solids, e.g. crystals, the value of r_a is not discrete but statistically distributed. Several transitions between energy levels are therefore possible and the original line spectrum becomes a band spectrum. If the pressure is increased even further, r_a becomes smaller and the width of the emission bands increases.
Fig. 17 shows that in addition to this, the central energy and the main wavelength of the emission band is also displaced.

2 Optical pumping

Optical pumping is a process in which light is radiated into a specimen under investigation and the effect of the light on the specimen is examined. It was in this way that the strange physical phenomenon was observed of atoms only being able to accept or release energy in well defined quantities. This observation led to the conclusion that atoms only have discrete energy states or energy levels. When light is absorbed or emitted, a transfer takes place between the energy levels (Fig. 18). A transition from the level with the energy E_1 to a level with the energy E_2 can occur if an incoming photon is absorbed with the energy $E_{Ph} = E_2 - E_1$. In the reverse case, a photon with the energy $E_{Ph} = E_2 - E_1$ is emitted if a transition of the atom takes place from a state with energy E_2 to one with energy E_1.

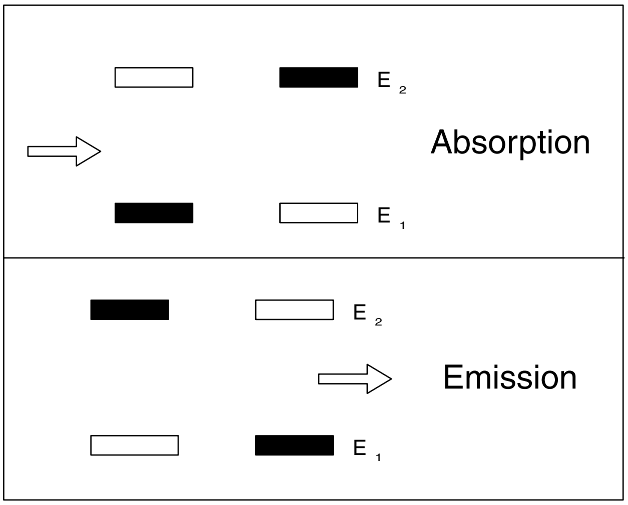


Fig. 18 : Absorption and emission

The two processes of absorption and emission differ in that an external field with the energy E_{Ph} must be present for absorption, whereas no field is present for emission. This emission occurs spontaneously from the system itself without external fields. It can be compared to the radioactive decay of an excited nucleus. The analogous inverse process to absorption, i.e. emission under the application of external fields is termed induced emission. For each of the processes the number of atoms can be stated which absorb or emit a photon per unit of time and per unit of volume.

$$\frac{dn_1}{dt} = -B_{12} \cdot n_1 \cdot u_{ph} \qquad \textbf{Absorption}$$

$$\frac{dn_2}{dt} = -B_{21} \cdot n_2 \cdot u_{ph} \qquad \textbf{Induced emission}$$

$$\frac{dn_2}{dt} = -A_{21} \cdot n_2 \qquad \textbf{Spontaneous emission}$$

B_{12} is the Einstein coefficient of absorption
B_{21} is the Einstein coefficient of induced emission and
A_{21} is the Einstein coefficient of spontaneous emission.
n_1 and n_2 are the densities of the atoms in the states 1 and 2 respectively, u_{ph} is the energy density of the external field. By integration of the equation for spontaneous emission, information is obtained on the variation of this type of emission with respect to time:

$$n_2(t) = n_2(t_0) \cdot e^{-A_{21} \cdot t}$$

A decay probability and a mean life-time can be given completely analogous to radioactive decay. The Einstein coefficient A_{21} represents this probability and

$$\tau = \frac{1}{A_{21}} \qquad \textbf{the mean life-time.}$$

This states the time which passes before the number of excited atoms has reduced to 1/e or before $n_2(t)$ has reached the value

$$n_2(t) = n_2(t_0) \cdot \frac{1}{e}.$$

For normal optical transitions, this value is between 10^{-8} and 10^{-9} sec. This life-time which is determined by the spontaneous transitions alone is relevant for the natural half width of a spectral line. According to the Heisenberg uncertainty principle, there is a relationship between the width and the life-time:

$$2\pi \cdot d\nu = \frac{1}{\tau} = A_{21}$$

where dν is the half-width of the spectral line.

According to the above requirements transitions will only take place if the energy of the absorbed or emitted photon is sharply defined, i.e.

$$E_{ph} = E_2 - E_1.$$

In actual fact nature is not that critical and it is seen that photons with a slightly different energy also take part in these processes. The reason for this is that the energy levels are broadened due to various mechanisms. Depending on how mobile the atoms are due to their temperature and how they are affected by interactions with their environment, there is a broadening dE which means that photons within this region are accepted. The width of the transition is given by the half width dE for the relevant transition (Fig. 19). The same theory is valid for emission.

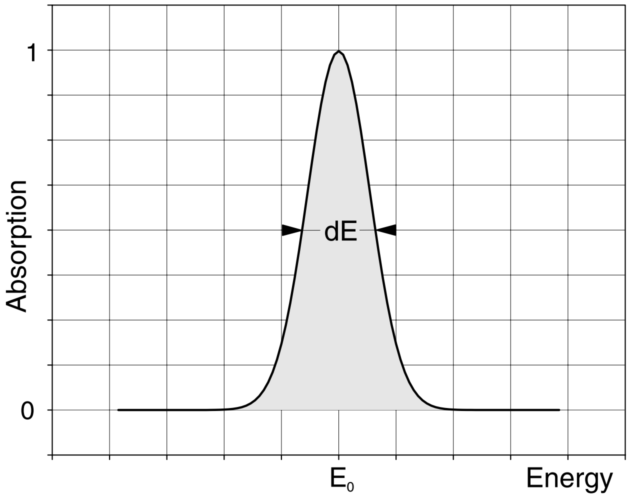


Fig. 19 : Broadened absorption transition.

E_0 is the energy at which the absorption is the highest. It corresponds to the value E_2 - E_1. For the sake of completeness it should be mentioned that there are also situations in which this value can be displaced. The shape of the absorption curve corresponds to a Gaussian distribution. By definition dE is the width of the curve at which the absorption has fallen to one half of the maximum value. This is known as the full width at half maximum (FWHM). If there are other transitions within the vicinity of this transition, they may overlap, producing a substantially wider absorption curve (Fig. 20). This is particularly important in the case of the absorption of laser diode radiation in Nd-YAG which is discussed later.

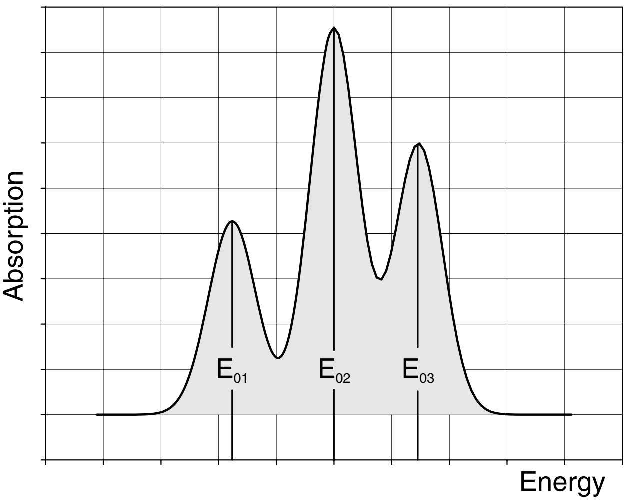


Fig 20 : Absorption for three neighbouring transitions with different absorption strengths

In principle an atom may have any number of energy levels, but they must be discrete. The transitions between the individual levels take place according to defined selection criteria. When the atom is excited with a defined energy, an emission spectrum is observed with characteristic energies and this spectrum gives precise information about the energy levels involved.

Excitation by optical pumping has therefore developed as a very important method used in spectroscopy. It is also an indispensable technique for the excitation of a number of different types of lasers. Optical pumping in conjunction with Nd-YAG lasers is of particular interest, because these have become widely accepted for industrial use, along with the CO_2 laser. The laser-active material which, in the case of the Nd-YAG laser, is excited by optical pumping, consists of Neodymium atoms that are accommodated in a transparent host crystal (YAG = **Y**ttrium **A**luminium **G**arnet).

Whereas up to a few years ago Nd-YAG lasers were almost excited using discharge lamps, optical pumping with laser diodes is becoming more significant. This is because laser diodes are available economically and they emit narrow band light at high optical powers, which matches the energy levels of the Nd-YAG crystal (Fig.21). The advantage over the discharge lamp is that the emission of laser diodes is nearly completely absorbed by the Nd-YAG, whereas the very wide spectral emission of discharge lamps is absorbed to only a small extent.

The efficiency of optical pumping with discharge lamps is about 3%, but figures of up to 50% can be achieved using laser diodes!

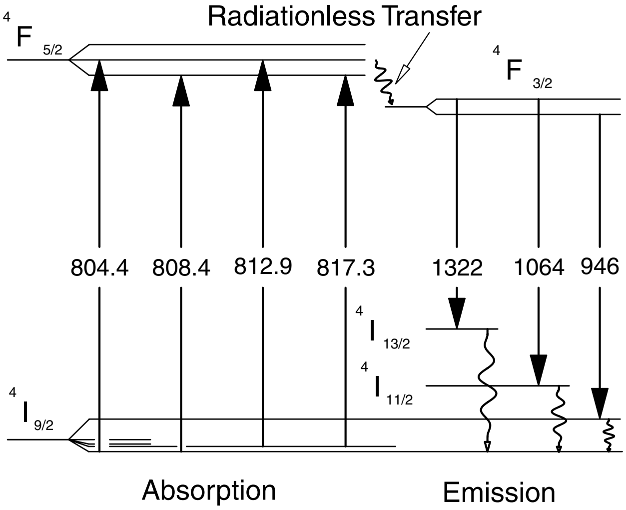


Fig. 21 : Relevant energy levels of Nd-YAG for optical pumping with laser diodes having wavelengths around 805 nm.

Some energy levels of the Nd atom are illustrated in Fig. 21. Here, only those are shown which are significant for optical pumping with laser diodes and which are important for the laser process. The levels are labelled with their spectroscopic notations. Since the Nd atoms are situated within the YAG host crystal, the otherwise degenerated energy levels of the isolated Nd atom split into a number of states.
This gives rise to the ground state $^4I_{9/2}$ from 5 substates and the state $^4F_{5/2}$, which can be pumped from 5 substates.

Since the wavelength of the pump-light source (diode laser) can vary within low limits, a total of three to four transitions can be pumped with high efficiency.

The Nd atoms of the $^4F_{5/2}$ state pass very quickly into the laser output level $^4F_{3/2}$. The laser transition which is technically most interesting takes place between the $^4F_{3/2}$ state as starting level and terminates in the $^4I_{11/2}$ state with an emitted wavelength of 1064 nm. From here the Nd atoms relax again into the ground state $^4I_{9/2}$ until the pumping process starts from the beginning again. The Neodymium therefore has an ideal four level system.

2.1 Four-level system of the Nd-YAG laser.

The principle is shown in Fig. 22. Under the radiation of a light field (optical pumping), transitions from ground state 1 to the upper level 4 occur. The reverse processes from state 4 to state 1 are prevented by very fast transitions from state 4 to state 3 without radiation.

The laser transition takes place from level 3 into level 2 which is thermally not populated. From here the Nd atoms relax again back to ground state 1.

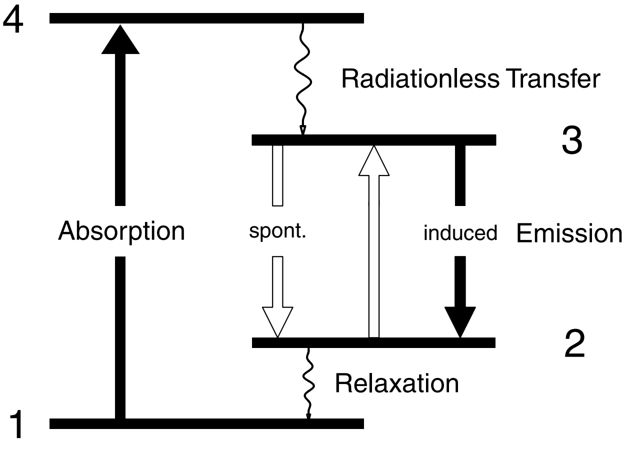


Fig. 22 : Principle of the four-level laser

W_{14}	probability of absorbing a pump photon.
S_{43}	probability of relaxation from state 4 to 3 .
S_{32}	probability of spontaneous emission of a photon.
W_{32}	probability of induced emission of a photon.
W_{23}	probability of induced absorption of a photon.
S_{21}	probability of relaxation from state 2 to state 1.

The irradiation by light, which leads to the population of an otherwise empty state, is termed optical pumping.
The emptying of a level occurs either with the emission of photons or without radiation. Transitions without radiation take place due to mechanical interactions such as collisions or vibrations and they are also designated as relaxation.
The number of transitions without radiation per second is termed the relaxation rate. Transitions in which photons are emitted occur spontaneously or are induced. Spontaneous transitions also occur without pumping processes. However, induced emissions only occur if a pumping process takes place. Rates are also stated here, one rate for spontaneous emission and another one for induced emission.
Each state which can interact with one or more other states is labelled with this type of rates. However, in Fig. 22 showing the principle, only the transition probabilities that are significant for the pump and laser processes are indicated. All the designated levels are populated to some extent due to pumping. The extent to which each state is populated is given by the number N_i of Nd atoms which are in the relevant state i of excitation:

State 1	N_1
State 2	N_2
State 3	N_3
State 4	N_4

Under the realistic assumption made in this example that the Nd atoms only pass through the labelled excitation steps, the sum of the population densities gives the Nd atoms which are available.

The desired laser oscillation will, however, only be achieved if an adequate population inversion can be established between states 3 and 2. The conditions under which laser emission occurs, together with how the laser behaves, can be predicted by a model of the so called rate equation model. Initially, the main interest will be focused on continuos laser operation.

2.2 Rate equation model for four levels

The model describes the situation in a simple but exact manner. Each of the levels involved is regarded as a reservoir to which or from which "particles" flow. The particles used in this picture represent the Nd YAG atoms related to their corresponding state. Particle streams flowing to the level are given a positive sign, those flowing away are given a negative sign. This is carried out for each of the involved states.

The number of excited atoms per unit of time in state 3 is:

$$\frac{dN_3}{dt}\Big|_p = \eta \cdot W_{14} \cdot N_1 = W_p \cdot N_1 \qquad \textbf{pump rate}$$

where η is the pumping efficiency. The transition from state 4 occurs so fast that the laser output level 3 is pumped immediately and the population N_4 density of state 4 is therefore $N_4 \sim 0$.

Spontaneous process

Another process affecting state 3 is spontaneous emission:

$$\frac{dN_3}{dt}\Big|_s = -\Gamma \cdot N_3 \qquad \textbf{spontaneous rate,}$$

where $\Gamma = 1/\tau_s$. τ_s is the mean life-time of a photon before it is spontaneously emitted.

Induced processes

Finally, the induced processes occurring between states 3 and 2 under the influence of the laser field must also be considered. The relevant rates are proportional to the difference in the population numbers N_2 and N_3 and to the photon density p of the laser field. The effective cross-section σ for the emission or absorption of a photon arises as a constant of proportionality:

$$\frac{dN_3}{dt} = \sigma \cdot c \cdot p \cdot \left(N_2 - N_3\right) \qquad \textbf{induced rate.}$$

Therefore the variation in the population density of level 3 with respect to time can be written as the sum of the separate rates:

$$\frac{dN_3}{dt} = \sigma \cdot c \cdot p \cdot \left(N_2 - N_3\right) - \Gamma \cdot N_3 + W_p \cdot N_1$$

Furthermore, the assumption is made that the transition from state 2 to state 1 is also so fast that only very few of the particles accumulate in state N_2 that means $N_2 = 0$ and the total number N_0 of Nd atoms is therefore:

$$N_0 = N_1 + N_3$$

Since N_0 is constant, also $dN_0/dt = 0$ and dN_1/dt becomes $-dN_3/dt$. Therefore the variation of the population density N_1 with respect to time is:

$$\frac{dN_1}{dt} = -\sigma \cdot c \cdot p \cdot \left(N_2 - N_3\right) + \Gamma \cdot N_3 - W_p \cdot N_1$$

It is important for the later laser process to know how the photon density on the laser transition 3 to 2 varies with respect to time. With each "induced" absorption process a photon is annihilated and a photon is created with each induced emission process.

$$\frac{dp}{dt}\Big|_i = -\sigma \cdot c \cdot p \cdot \left(N_2 - N_3\right) \qquad \textbf{induced}$$

Once created, the photon density does not remain in a resonator, instead it reduces with the time constant τ_{ph}, because photons are leaving at the mirrors of the resonators or are lost in other ways.

$$\frac{dp}{dt}\Bigg|_l = -\frac{p}{\tau_{ph}} \qquad \textbf{losses}$$

The complete variation of the photon density with respect to time is:

$$\frac{dp}{dt} = \sigma \cdot c \cdot p \cdot \left(N_3 - N_2\right) - \frac{p}{\tau_{ph}}$$

For simplification the population inversion N_3-N_2 is designated as n . The variation of the population inversion with respect to time is obtained by the following relations:

$$\frac{dn}{dt} = -\sigma \cdot c \cdot p - \Gamma \cdot n + W_p \cdot \left(N_0 - n\right) \qquad \textbf{(2.2.1)}$$

and for the photon density:

$$\frac{dp}{dt} = p \cdot (\sigma \cdot c \cdot n - \frac{1}{\tau_{ph}}). \qquad \textbf{(2.2.2)}$$

2.2.1 Solution of the rate equations

The differential equations (2.2.1) and (2.2.2) form a pair of coupled conditional equations for the two unknown functions n(t) and p(t). The equations are non-linear because they both contain the term ***pn***. Analytical solutions are not known and one has to rely on computerised solutions.

2.2.2 Steady-state solution

When the system is in the state of equilibrium, i.e. for steady-state laser operation, the values for dn/dt and dp/dt are equal to zero. In this case an expression for the population inversion is obtained immediately:

$$n = \frac{N \cdot W_p}{\sigma \cdot c \cdot p + W_p + \Gamma}$$

When the laser is operated below or just at the threshold, no photon field is formed *(p=0)*. In this case $Wp << \Gamma$ and the threshold inversion is given by:

$$n(p=0) = n_0 = N \cdot \frac{W_p}{\Gamma}$$

This equation states that in a four-level laser an inversion immediately is produced when it is pumped. This is a particular advantage as opposed to other laser systems. Unfortunately, neither the photon density nor the pumping rate are directly accessible by measurement. However, the photon density is coupled to an easily measured quantity, i.e. the power applied in the pumping process. If the relationships between the photon density p and the corresponding intensity, as well as the resonator output and loss characteristics are considered, the output power P_a of a four-level laser can be obtained as:

$$P_a = \eta \cdot \frac{E_{32}}{E_{41}} \cdot (P_p - P_{th}) \cdot \frac{T}{T+L} \qquad \textbf{(2.2.3)}$$

In this equation E_{32} signifies the energy difference between states 3 and 2 (laser wavelength). E_{41} is the energy difference between states 4 and 1 (pump wavelength), T is the transmission of the output coupling mirror, L is the loss in the resonator due to dispersion, absorption or refraction. P_p is the pump power and P_{th} is the threshold pump power. Above the threshold pump power P_{th} the output power of the laser increases linearly with the pump power. The slope α_S of the straight line (Fig. 23) is one of the most important parameters of a laser and is termed the slope efficiency.

$$\alpha_S = \eta \cdot \frac{E_{32}}{E_{41}} \cdot \frac{T}{T+L} \qquad \textbf{(2.2.4)}$$

The quantity E_{32} / E_{42} is also known as the quantum efficiency. It gives the energy ratio of a laser photon to the pump photon. For the Nd-YAG laser pumped by a diode laser this is, for example, 810 nm / 1064 nm = 0,76. The value η is the quantum yield, but unfortunately sometimes both quantities are commonly named as the quantum efficiency.

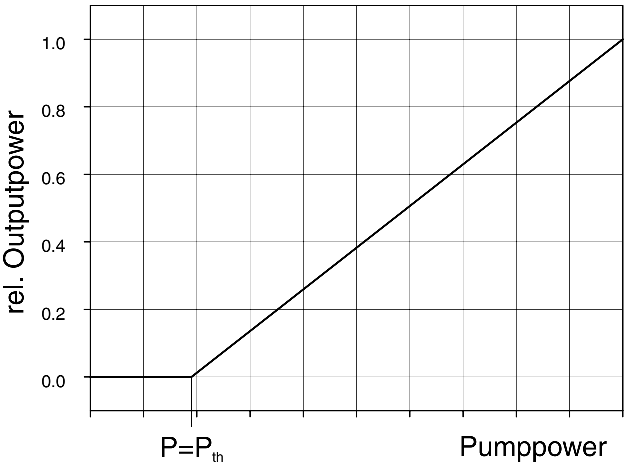


Fig. 23 : Laser output power as a function of the pump power

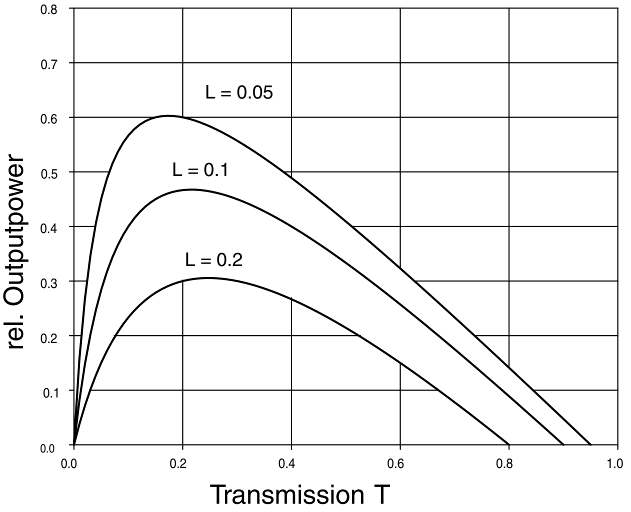


Fig. 24 : Laser output power in dependence of the transmission T of the output mirror and the losses L

For the laser designer it is important to obtain the highest possible output at the highest possible efficiency. Another important feature is that the transmission T for the resonator output mirror must be selected as large as possible according to equation 2.2.4. However, this has the consequence that the threshold pump power P_{th} increases and the output power decreases according to equation 2.2.3. A compromise between equations 2.2.3 and 2.2.4 must therefore be found. In practice the losses L depend on various parameters of the resonator including the quality of the laser rod, the absorption losses of the laser mirror etc., so that a mathematical formula covering all effects would be too complicated. It has proven useful to measure the curve of Fig. 24 directly at the laser to find the degree of output coupling. A series of output mirrors with various transmission values is used for this purpose.

2.2.3 Time-varying solution of the rate equations

The previous solutions described the situation where the laser operates in a steady state. However, for practical operation of the laser, conditions in disturbed equilibrium must also be considered. These kinds of disturbances occur when the intensity of the pump-light source change or the laser resonator is slightly disturbed mechanically. Large deviations from the steady state are particularly important when they cause problems (e.g. spiking and hot spots), but also when they lead to useful operating modes (Q switching). Small deviations of the steady state with $\delta n << n$ or $\delta p << p$ lead to damped harmonic oscillations of n and p. Larger deviations produce undamped non-harmonic oscillations. In this case, power peaks (spiking) of such a intensity may occur that the laser mirrors or the Nd-YAG materials can be destroyed. However, if disturbances are carried out in a controlled manner, these types of power peaks, which extend up into the gigawatt region (!), can be used to advantage. Computerised solutions must be used in calculating the solutions to the rate equations for these cases. In the following, these cases are therefore only qualitatively described.

2.2.3.1 Spiking

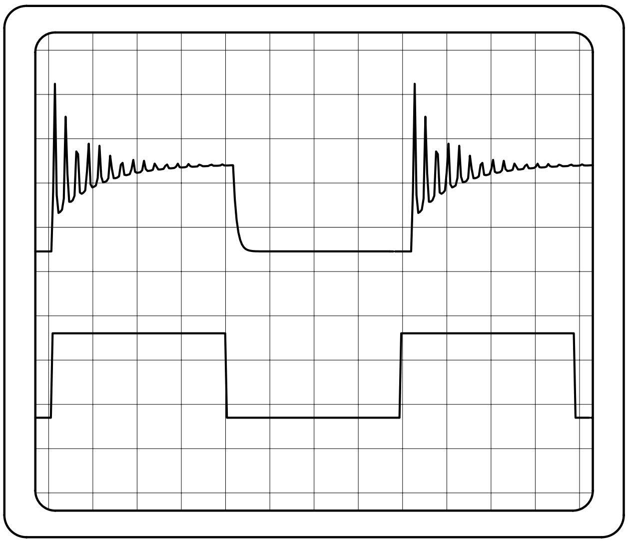


Fig. 25 : Spiking of the Nd-YAG laser

A large deviation from the steady state undoubtedly occurs when the laser is switched on or when the pump-light source is switched on. Until the threshold pump power P_{th} is reached, there are practically no photons present in the resonator.

When the population inversion reaches the threshold, a photon field is formed. However, due to the resonator propagation time, it takes a while until the photon density reaches the steady state value. During this period the inversion, which rises linearly with time, increases above the value of the threshold inversion.

This in turn means a more rapid increase in the photon density. This rise is so rapid that the inversion falls to a value slightly below the threshold and the laser oscillation stops (Fig. 25). The process starts again, but this time the laser is only slightly below the threshold and the expected inversion overshoot is not so large as before. In this manner the system approaches the steady state.

The first power spike (initial spike) can reach a peak power of a factor 100 to 1000 higher than the steady state power value. Spiking therefore can cause serious problems and it can lead to the destruction of the optical surfaces so that the laser might destroy itself during switch-on. This behaviour which will be observed in the latter experiments indicates, that the Nd YAG crystal can store energy.

2.3 The pump-light source

High power Nd-YAG lasers are still mainly pumped with discharge lamps. Commercially available laser systems can output up to 2,000 watts of continuous-waves laser power. If one bears in mind that the overall efficiency of the Nd-YAG laser is about 1 - 2%, then the discharge lamps must have a light output power of approximately 100 - 200 kW. From the light produced, only 2,000W is converted into laser power and the rest appears as heat which must be extracted using complicated cooling systems.

The reason for this "poor" efficiency is that the light produced by the discharge lamps has a broad spectral distribution and the Nd-YAG crystal can only accept the offered light in a number of narrow absorption bands.

Up to the present time it has not been possible, in spite of complex and intensive research, to develop discharge lamps which have an emission tuned to the absorption bands of the Nd-YAG crystal.

Along these lines, the gas, which is excited in the discharge lamps, has been doped with additives to produce preferred spectral emissions.

Laser diodes do not have this disadvantage since they emit intensive laser light in a narrow spectral range of only a few nanometres. The wavelength of diode emission therefore matches an absorption band of the Nd-YAG crystal very well. It is possible to achieve efficiencies of 50-80% in this manner.

However, there are not at present any laser diodes available with output powers greater then 10 W. On account of the attractive features of laser diodes which, in contrast to discharge lamps, do not require any heavy duty power supplies for high voltages (approximately 1000V), intensive research has started in the manufacture of high-power laser diodes.

A further advantage of laser diodes is their very small size which enables a large number of individual diodes to be integrated on one common chip. Rows of pump-light sources with optical output powers into the kW region can be built up with this type of laser diode arrays.

The laser diodes are a special class of lasers. They differ from "conventional“ lasers in two points: Firstly for the classical lasers the laser-active atoms (molecules or ions) are independent of one another and only the same energy levels are used for the laser process.

This means in principle that in order to produce a population inversion an infinite number of atoms can contribute

(Boltzmann statistics). This is not the case with semiconductor lasers. Here a defined energy level can only be occupied by two active particles (electrons, Pauli Principle). But in semiconductors, the wave functions of the individual atoms overlap to form a common energy band and the extent to which the level is occupied follows the Fermi Dirac statistics.

When considering the laser process, the transition between the distribution of population in two energy bands instead of two energy levels must be taken into account as for conventional lasers.

The second important difference concerns the propagation of the laser light within the pn zone. The spatial intensity distribution of the laser beam is defined by the laser medium and not by the resonator as for normal lasers.

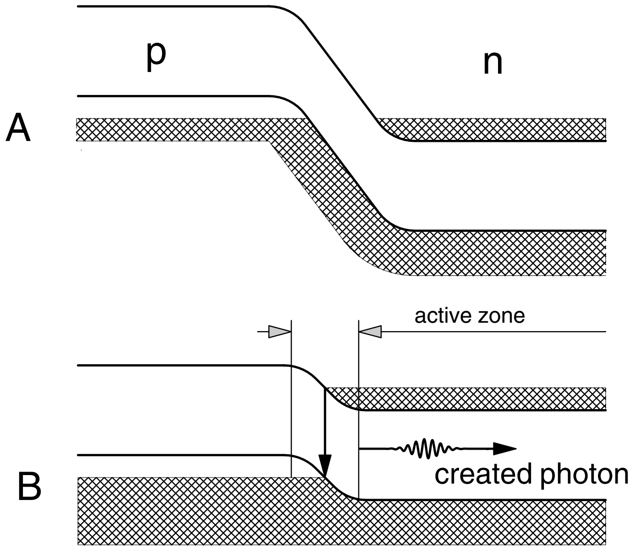


Fig. 26 : pn junction without applied voltage (A), with applied forward voltage (B) . The active zone contains both electrons and holes which produce a photon when recombining.

These two points lead to the fact that the beam characteristics and the spectral properties of semiconductor lasers are significantly different from those of conventional lasers:

For 1.) Laser diodes do not have any inherently defined emission wavelength, because there are no two discrete energy levels that are responsible for the laser process as with traditional lasers, but rather an energy distribution of electrons in energy bands.

For 2.) The production and guidance of the laser light takes place in a very narrow space (pn layer) (Fig. 27). In contrast to the conventional laser, the dimensions of the resonator are about the same order of magnitude as the wavelength of the laser beam. The spatial distribution of the laser beam and the mode structure is defined by waveguides, whereas the light is freely propagating within a resonator of a conventional type of laser. These two points influence the application of laser diodes.

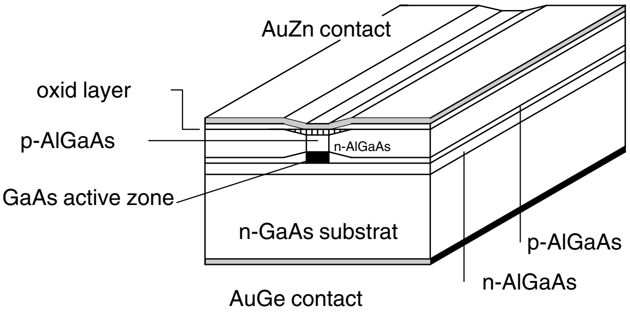


Fig. 27 : (GaAl)As semiconductor laser with double hetero structure and single stripe geometry

Before the laser beam from laser diodes can be used in the usual manner, the strong divergence must be corrected by sometimes complex optical systems. Also, the corrected parallel beam does not have a round cross-sectional shape, but is elliptical and sometimes almost rectangular. The corrections required to the beam of a laser diode and the difficulties in obtaining the required focusing characteristics with comparable power densities mean that the expense involved in the optics obviates the cost advantages of laser diodes.

Because of this, a way is sought of using the laser diode, not as a primary high power laser, but instead, due to its excellent other characteristics, as a pump-light source for conventional laser systems. It should also be mentioned that the above arguments are only relevant when considering laser systems with high powers for applications in material processing.

Without a doubt, laser diodes are excellent as primary lasers in communication techniques. Fig. 27 shows a diagrammatic representation of a (GaAl) As semiconductor laser with double hetero-structure and single stripe geometry. Charge carriers are injected into the very thin (approx. 0,2 μm) active layer by applying a voltage via the upper AuZn contact stripe which is only a few μm wide. The active zone is embedded between two hetero-junction boundaries which act as barriers for the charge carriers. If the flow of current is high enough, population inversion is formed in the active volume.

The laser beam leaves the active zone through the exit window. The crystal has such a high index of refraction that the end surfaces have a sufficient degree of reflection so that no further coating is required and they therefore act as laser resonator mirrors.

There are a large number of types of diode lasers which differ in their structure. Only multiple stripe laser diodes, known as diode arrays, are available at present as high-power laser diodes with powers from 3 W upwards. Single stripe diodes (Fig. 27) are currently limited in their output power to about 3 W. Output powers of up to 100 W can be achieved by rows of a number of active zones on one chip. The closer the separation between the stripes, the lower the laser threshold will be. With a sufficiently high gain "super modes" are created which correspond to one oscillation state for the coupled active zones. The beam profile of the super mode is dispersed according to the extent of the stripe. A laser diode with a single stripe and an output power of 500 mW is used for the Nd-YAG experimental laser.

A further characteristic of the diode laser is the strong dependence of the laser wavelengths on the temperature of the semiconductor laser (app. 0.25 nm/°K) and of the injection current (app. 0.05 nm/mA). Users who need a defined wavelength must maintain the temperature and the injection current constant to the required values.

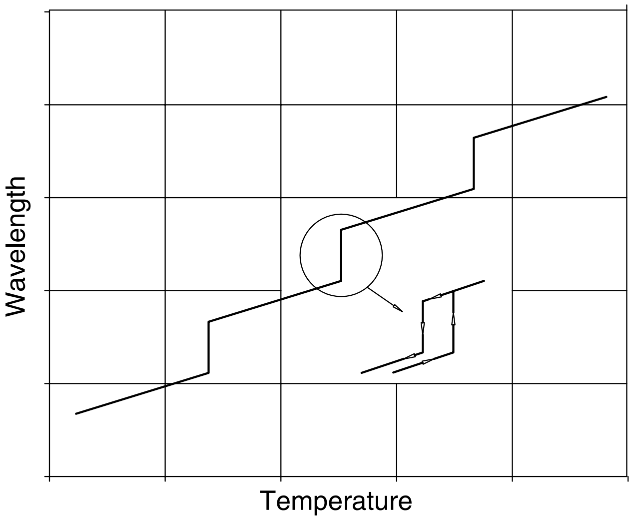


Fig. 28 : Emission wavelength in dependence of the laser diode crystal temperature showing a hysteresis

The wavelength of the laser beam can be changed by altering the temperature. The wavelength increases with rising temperature. This is due to the fact that with rising temperature the refractive index and the length of the active zone, and therefore of the resonator, are increased. Above a certain temperature the mode no longer fits into the resonator and a different one oscillates for which the conditions are now more favourable.

Since the mode interval of the extremely short resonator (typically 300 µm) is very large, the mode jump is about 0.3 nm. If the temperature is reduced, the laser jumps back in its wavelength again. The return jump does not necessarily take place into the initial mode.

Applications which rely on correct variable tuning of the wavelength of the laser diode should be carried out in a region of the curve in Fig. 28 that is remote from any jumps. Similar behaviour is also observed when the injection current, and therefore the laser output power, is varied. Here the change in wavelength arises mainly due to the increase in the refractive index which in turn is due to the rise in charge carrier density in the active zone.

With higher output power the temperature also rises due to losses in the form of heat in the active zone. The dependence of the current, and therefore the output power, on the temperature is typical for semiconductors (Fig. 29). It is therefore essential that in practical operation the beam power is continually monitored.

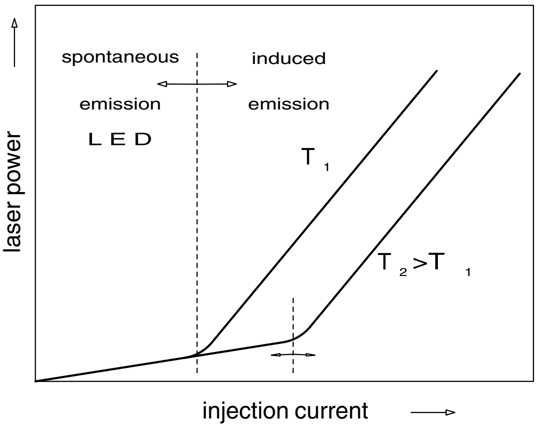


Fig. 29 : Dependence of Laser power on the injection current with the temperature as a parameter.

To achieve this, a photodiode is built into the laser housing and the photodiode supplies a signal which is independent of temperature and which is proportional to the beam power. This signal is used as input for a control circuit with which the laser current is corrected and laser power maintained constant. The wavelength of the laser diode can be expressed as a function the two variables temperature T and injection current I in the following equation:

$$\lambda(T,I)=\lambda(T_0,I_0)+\alpha_T\cdot(T-T_0)+\alpha_I\cdot(I-I_0) +\alpha_T^2\cdot(T-T_0)^2+\alpha_I^2\cdot(I-I_0)^2 +\ldots+\alpha_T^n\cdot(T-T_0)^n+\alpha_I^n\cdot(I-I_0)^n$$

For practical considerations it is sufficient to use only the linear terms of the above equation. This approximation is good for $\delta\lambda/\lambda > 10^{-6}$. For operation at constant wavelength, $\lambda(T,I)$ = const. = λ_c , the equation can be written as:

$$I=I_0+\frac{1}{\alpha_T}(\lambda-\lambda_0)+\frac{\alpha_T}{\alpha_I}\cdot(T-T_0)$$

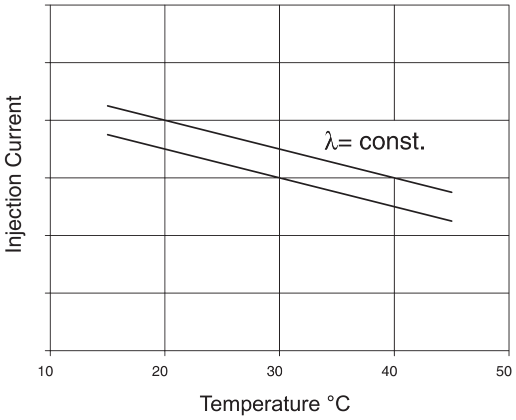


Fig. 30 : Injection current versus temperature for constant wavelength, shown for two different wavelength

Once this curve is measured as it will be done later, the important coefficients α_T and α_I can be determined.